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Femtosecond Optical Pulse Propagation through the Single Resonance Lorentz Model Dielectric

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Abstract: *We propose to apply elementary wave packets (EWP) series to investigate femtosecond (fs) optical pulse propagation through the single resonance Lorentz model dielectric. Ortho – normalized EWP – functions on confine segment are defined as real ones. So we use real functions solution of wave equation. Then we study well-known "precursory" for physically grounded samples of fs optical pulses. That enables us to propose formula for optical data transmission velocity (ODTV). Its value does not exceed light velocity in vacuum.*

Keywords: *dispersive medium, femtosecond pulse, elementary wave packets series, signal velocity*

1. Introduction

Ultrawideband optical signal propagation through the Lorentz model dielectric have been studied for tens years, especially, the evolution of precursor pulse. The origins of this problem date back to Sommerfeld [1] and Brillouin [2]. Oughstun [3] has updated and expanded this early results by applying modern asymptotic techniques Authors of references [1],[2] studied propagation of a sinusoidally modulated step signal. Modern publications [3],[4] were devoted to study propagation of restricted model of optical pulse: step-function, δ - pulse, monochromatic, Van Bladel envelope pulsefunction. Elementary wave packets series (EWP- functions) [5] represents complete ortho-normalized system that fits some essential physical properties of pulse electromagnetic signals. That enable us to study precursor formation for fs optical pulses. In particular we take into consideration precursor duration when designed optical data transfer velocity (ODTV) formula.

2. Solving Wave Equation for Dispersive Medium

We seek real-valued solution for scalar linear wave equation in nonmagnetic $(\mu=1)$ dispersive medium with boundary conditions

$$\frac{\partial^2 E(z,t)}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial^2 P(z,t)}{\partial t^2}, \tag{1}$$

where $E(z,t)$ - electric component, $P(z,t)=\int\limits_0^\infty \chi(t')E(t-t')dt'$ -polarization,

$E(0,t)=E_0(t)$ -input optical pulse, $\frac{\partial E}{\partial z}(0,t)=E_1(t)$ - boundary condition.

Equation (1) in partial derivatives can be simplified by Fourier – integral decomposition. As a result one can get equation with total derivative

$$\frac{\partial^2 E(z,\omega)}{\partial z^2}-\frac{\omega^2}{c^2}\varepsilon(\omega)E(z,\omega)=0, \tag{2}$$

$\varepsilon(\omega)=1+4\pi\chi(\omega)$ - dielectric susceptibility. Then apply boundary conditions for $E_r(z,\omega)$ and finally we get

$$E(z,t)=\sqrt{\frac{2}{\pi}}\int\limits_0^\infty e^{-\kappa\frac{\omega}{c}z}Cos[\omega t]\left(E_0(\omega)Cos\left[n\frac{\omega}{c}z\right]-\frac{E_1(\omega)}{\omega}Sin\left[n\frac{\omega}{c}z\right]\right)d\omega, \tag{3}$$

where $E_0(\omega)=\sqrt{\dfrac{2}{\pi}}\int\limits_0^T E\big|_{z=0} Cos[\omega t]\,dt,$ $\qquad E_1(\omega)=\sqrt{\dfrac{2}{\pi}}\int\limits_0^T \dfrac{\partial E}{\partial t}\bigg|_{z=0} C[\omega o]$ ş

T - duration of the input optical pulse.

3. LORENTZ-LORENZ MODEL FOR DISPERSIVE MEDIUM

Before passing to calculation of the optical pulse spreading through the dispersive medium, let us briefly examine Lorentz model. Despite great age of the theory, this model stay in order thanks to their simplicity and acknowledgement among experimenting staff [6]-[7]. Along with basic offering of medium as ensemble of noninteracting oscillators, we take into account difference between average $\bar{E}(t)$ and acting $\bar{E}_a(t)$ amplitudes

$$\bar{E}_a(t)=\bar{E}(t)+\frac{4\pi}{c}\bar{P} \tag{4}$$

This permits us to investigate not only rare medium. As it is shown, for example, in [6]

$$\varepsilon(\omega)=1+\frac{\omega_p^2\left(\tilde{\omega}_0^2-\omega^2\right)}{\left(\tilde{\omega}_0^2-\omega^2\right)^2+\omega^2 y^2}+ \\ +i\frac{\omega_p^2 y\,\omega}{\left(\tilde{\omega}_0^2-\omega^2\right)^2+\omega^2 y^2}, \tag{5}$$

Where $\omega_p^2=4\pi Ne^2/m,$ $\tilde{\omega}_0^2=\omega_0{}^2-\omega_p{}^2/3,$ y - damping coefficient, N - concentration of oscillators. The last step is introduction of $n(\omega),\kappa(\omega)$

$$n(\omega)+i\kappa(\omega)=\sqrt{\varepsilon(\omega)}=\sqrt{\varepsilon'+i\varepsilon''}$$

Then

$$n(\omega)=\left(\left(\left(\varepsilon'^2+\varepsilon''^2\right)^{1/2}+\varepsilon'\right)/2\right)^{1/2}, \tag{6}$$

$$\kappa(\omega)=\left(\left(\left(\varepsilon'^2+\varepsilon''^2\right)^{1/2}-\varepsilon'\right)/2\right)^{1/2}, \tag{7}$$

where

$$\varepsilon'=1+\frac{\omega_p^2\left(\tilde{\omega}_0^2-\omega^2\right)}{\left(\tilde{\omega}_0^2-\omega^2\right)^2+\omega^2 y^2}, \tag{8}$$

$$\varepsilon''=\frac{\omega_p^2\omega y}{\left(\tilde{\omega}_0^2-\omega^2\right)^2+\omega^2 y^2} \tag{9}$$

Formulae (8), (9) can be normalized : $\omega_p\equiv\omega_p T,$ $\quad y\equiv yT,$ $\quad \omega_0\equiv\omega_0 T.$ So, equation (3) together with (6)-(7) and parameters of input pulse: $E_0(\omega),\quad E_1(\omega)$

form the solution for optical pulse, which propagates through the dispersive medium. Next part will be devoted to investigation of particularities of EWP-pulses propagation.

Typical normalized frequency dependencies for $n(\omega),\ \kappa(\omega),\ E_0(\omega)$ are shown in Fig.1 accordingly

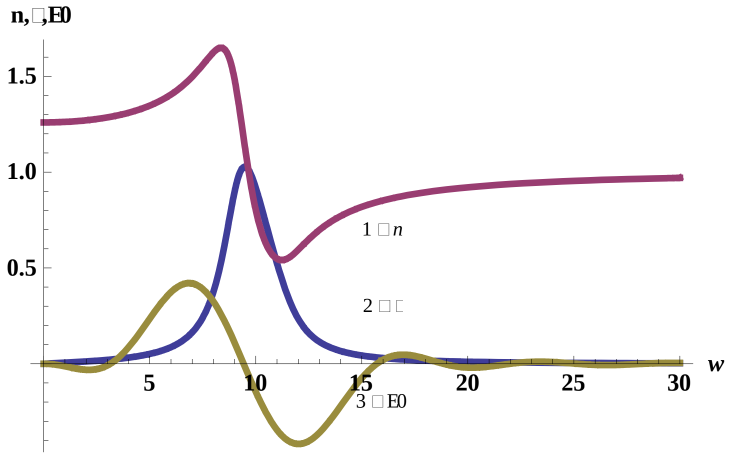


Figure1. *Refractive index* $n(\omega)-1$, *absorption coefficient* $\kappa(\omega)-2$, *signal spectrum* $E_2(z,\omega)-3$ *:* $\omega_p=7, \omega_0=10, \quad y=2.$

4. EWP–PULSE PROPAGATION THROUGH THE ONE – RESONANCE DISPERSIVE MEDIUM

In this part we study EWP – pulse of 3–rd order

$$E_6(t)=(6)^{-1/2}\left(Cos[2\pi t]+Cos[4\pi t]+Cos[6\pi t]-3Cos[8\pi t]\right), 0\le t\le 1, \quad A=1. \tag{10}$$

The CosFourier Transform of pulse shape (4) is

$$E_0(\omega)=-\frac{8\pi^{3/2}\omega}{\sqrt{3}}\times\frac{\left(5408\pi^4-700\pi^2\omega^2+17\omega^4\right)\left(Sin[\omega]\right)}{\left(4\pi^2-\omega^2\right)\left(16\pi^2-\omega^2\right)\left(36\pi^2-\omega^2\right)\left(64\pi^2-\omega^2\right)}, 0\le\omega\le\infty,\ \omega\equiv\omega T,\ t=t/T \tag{11}$$

$$E_1(\omega)=\frac{8\pi^{3/2}\omega^2}{\sqrt{3}}\times\frac{\left(5408\pi^4-700\pi^2\omega^2+17\omega^4\right)\left(-1+Cos[\omega]\right)}{\left(4\pi^2-\omega^2\right)\left(16\pi^2-\omega^2\right)\left(36\pi^2-\omega^2\right)\left(64\pi^2-\omega^2\right)}, \quad 0\le\omega\le\infty, \tag{12}$$

Let us investigate time and frequency dependencies for pulse with distance z, when resonant frequency of matter ω_0 is near the maximum of function $E_0(\omega)$: $\omega_0 \approx 25$. Dependencies for $n(\omega)$, $\kappa(\omega)$, $E_0(\omega)$ are shown below (Fig.2). Part of the spectrum $(\omega<\omega_0)$ has $n>1$, another part $(\omega>\omega_0)$ has $n<1$, part $(24\le\omega\le 26)$ is region of anomalons dispersion $(dn/d\omega<0)$. Pulse envelope, its position, spectrum depend upon distance z and medium parameters: ω_p, ω_0, y.

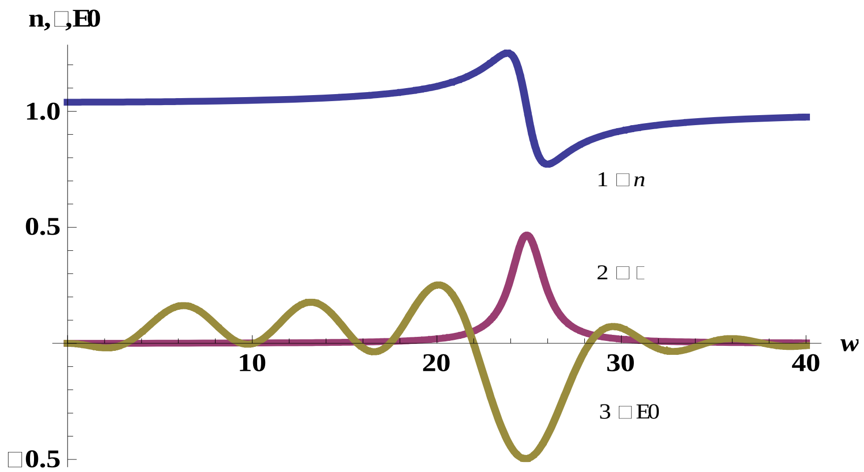
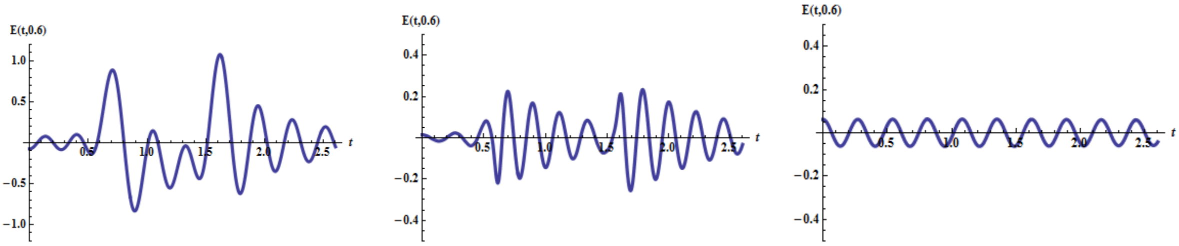


Figure2. *Refractive index* $n(\omega)-1$, *absorption coefficient* $\kappa(\omega)-2$, *signal spectrum* $E_6(\omega)-3$ *:* $\omega_p=7, \ \omega_0=25, \ y=2.$

To understand pulse propagation in details let us investigate: 1) behavior of low-frequency spectrum and high-frequency one; 2) parameters of so-called “forerunner”; 3) velocity of the EWP-pulse. In

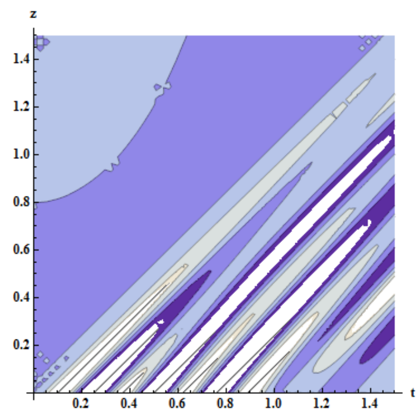
Fig. 3 influence of high, middle and low frequencies are shown. First of all, EWP-pulse is forming by the interference of spectral components of all three parts. Low frequencies components brings in the basic energy in pulse Fig.3a. High-frequencies execute action as shot pulse for resonant contour (Fig.3b).



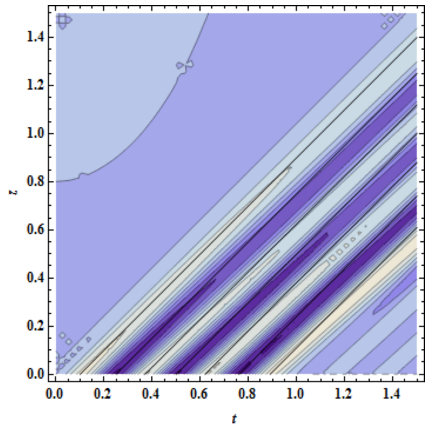
a) b) c)

Figure3. *Spectrum decomposition of signal E6 (t , z) for* $z = 0.6,$ *and frequency band* $\left\{\omega^{\min}, \omega^{\max}\right\}$ *: (a) –* $\left\{0, 23\right\}$; *(b) -* $\left\{26, \infty\right\}$; *(c) -* $\left\{23, 26\right\}$; $\omega_p = 7,$ $\omega_0 = 25,$ $y = 2.$

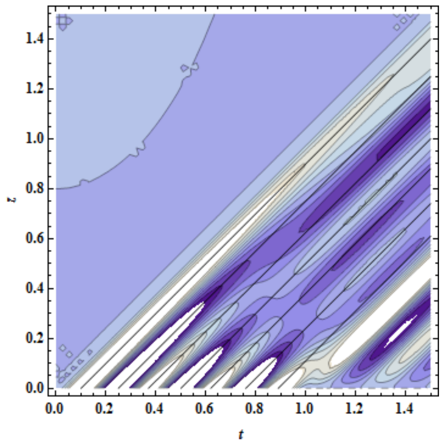
After fore front of the pulse and back front there are relaxation oscillations. The duration of the pulses front $t_p \approx 0.1,$ $2\pi / \omega_0 \approx 0.25.$ Anomalous dispersion region must have "anomalous velocities". One can see (Fig.3c) that local change of amplitude with distance z takes place almost momentary. But we can not definite velocity for the "part of the pulse". For this purpose let calculate contour dependencies $E(z,t) = C_i.,$ C_i -constants. We do it for two medium frequency ω_0 disposals: 1) $\omega_0 = 20 < 25,$ 2) $\omega_0 = 30 > 25.$ Corresponding dependencies are shown in Fig. 4. for different $\omega_0 : (a) - 20; (b) - 30; (c) - 24 : \omega_p = 7, y = 2.$ Dark lines conform to light propagation in vacuum.



a)Time



b)time



b)time

Figure4. *Contour dependencies* $E_6(z,t) = C_i.$

At the same time we draw up lines, which are correspondent to light velocity in vacuum $\left(C = 3\cdot 10^8 m/s\right)$. One can see, that peaks of the pulse have velocity that is less then C for $\omega_0 > 25$, and more then C for $\omega_0 < 25$. These results corresponding to the conception of group velocity [6], [8]. Properly from dependencies $E(z,t) = C_i$. for $\omega_0 = 24$, the velocities of peaks propagation equal to $\approx C$. As it was stated by Sommerfeld and Brillouin [1], [2], the so called 'forerunner" is proceding the basic pulse.

5. C - RESTRICTION ON THE DATA TRANSFER VELOCITY

In consideration of causality the only optical data transfer velocity (ODTV) is restricted with light constant C in vacuum. This is agreed – upon point of view majority of researches. As for phase and group velocities they may take any value. For example there are so called "slow light" and "fast light" [9]-[11]. However these notions appeared as the consequence of slowly varying amplitude approximation (or quasi monochromatic approximation) for electromagnetic signals [12].

In this chapter we propose quantitative definition of velocity of information spreading in dispersive medium (V_d). It has to take into account: 1) time delay of signal due to propagation; 2) duration of the signal; 3) modification of information value; 4) time delay for signal due to "precursor" beforunuing. Flow J (z) of information on the z-distance is

$$J(z) = \frac{z}{z/C + T_p + T}\log\left(1 + \frac{S(z)}{N}\right)/CTS \qquad (13)$$

where - $S(z)/N$ - signal to noise ratio (SNR), T_p - duration of precusor pulse, T - duration of signal, S - square of the optical beam.

The flow (13) is equal to volume concentration of the information on the entrance multiplying on ODTV which depends upon the distance $V_d(z)$. Then

$$\frac{V_d(z)}{C} = \frac{z/C}{z/C + T_p + T}\log\left(1 + \frac{S(z)}{N}\right)/\log\left(1 + \frac{S_0}{N}\right) \qquad (14)$$

One can see that $V_d(z) < C$, because both of two multipliers are less then 1. Now let us calculate $T_p(z)$ and $S(z)$. Basic signal follows by precursor. Then time interval $\left[T_p, T_p + T\right]$ has to fall in the time interval of orthogonality of signal. So

$$I(z,T_p) = \int_{z+T_p}^{z+T_p+1} E(z,t)dt \approx 0 \qquad (15)$$

The result of numerical calculation of equation (15) is shown in Fig.5. Because $T_p << T$, then we can use linear approximation $T_p \approx p\,z, \quad p << 1$.

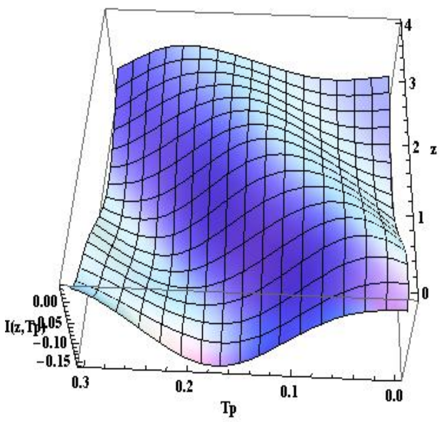


Figure5. *Duration of precursor pulse* $T_p(z)$: $\omega_p = 7, \omega_0 = 30, \ y = 2$.

Now we can calculate $S(z)/S_0$. Let us use orthogonality for the time interval $\left[T_p, T_p+1\right]$.

Then

$$\alpha(z)\int\limits_{z+T_p}^{z+T_p+1} E_0^2(t)dt = \int\limits_{z+T_p}^{z+T_p+1} E(t,z)E_0(t)dt, \qquad \alpha(z) = \int\limits_{z+T_p}^{z+T_p+1} E(t,z)E_0(t)dt \tag{16}$$

So equation (14) can be written taking into consideration (16) (for normalized parameters)

$$\frac{V_d(z)}{C} = \frac{z}{z+T_p(z)+1}\frac{\log\left(1+\alpha(z)\frac{S_0}{N}\right)}{\log\left(1+\frac{S_0}{N}\right)} \tag{17}$$

Result of calculation for $\alpha(z)$ is shown in Fig.6. Falling down of $\alpha(z)$ with distance growing is due to dispersion of the medium: absorption and signal shape degradation.

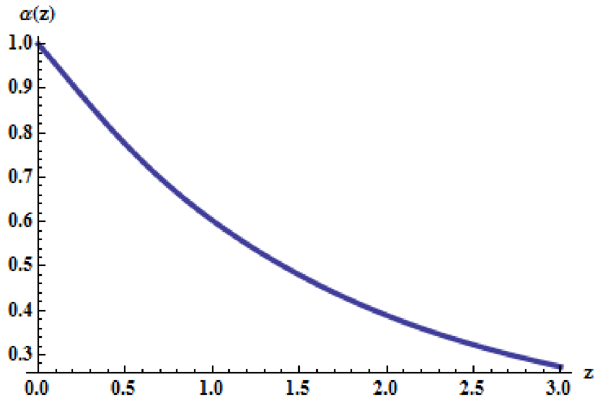


Figure6. *Weight coefficient* $\alpha(z)$ *for* $E_6(t)$ *EWP – pulse:* $\omega_p = 7,\ \omega_0 = 30,\ y = 2.$

6. Conclusions

We have used for the investigation of optical pulse propagation proposed Elementary Waves Packets (EWP) series for representation of electromagnetic signal of any duration [5]. EWP base functions are superwideband signals. So introducing superwideband signals cause the necessity to investigate its propagation through the dispersive medium.The characteristic property of this problem is considerable excess of the spectral width of the optical signal above spectral width of medium oscillator. Therefore we investigate “high-frequency” and “low-frequency” oscillator apart. Our calculations confirm the existence of so called “precursor pulse’ that propagates through the medium with velocity C of the light in vacuum. The duration of the precursor T_p is equal to delay of the orthogonal time interval T for electromagnetic pulse. The second peculiarity of the pulse is - high frequencies of the pulse act on medium like impact to extite relaxation oscillations at the leading and trailing limits of the optical pulse. As regards to velocity of light propagation, we examine so called optical data transfer velocity (ODTV). As it is generally assume, only information velocity is restricted by light-velocity C in vacuum [8],[10],[11]. So we proposed quantitative definition of ODTV. It takes into ascount time delay of pulse propagation in dispersive medium and information losses. It is obviously that ODTV in vacuum is equal to C.

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