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How Far Away and Often from Bohr Orbit

(Treatise on the Choice)

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Abstract: *In this paper according to Heracletean dynamics the electron position on Bohr orbit is recognized as the most probable one amongst 274 discrete ground state positions. Consequently the probability of finding the electron in the proposed positions is calculated.*

Keywords: *Heracletean dynamics and probability of finding the electron on ground state orbits of Hydrogen atom*

1. PREFACE

The subject of interest of this paper is with the help of Heracletean dynamics[1] on double surface[2] to explain how the electron could leave Bohr orbit without changing the total energy and how often that event could be done.

2. BOHR ORBIT

On Bohr orbit in the ground state of Hydrogen atom the electron possesses due to equality of Coulomb and centripetal force the lowest and therefore most favourable total energy $W_t = -\frac{m_e c^2}{2\alpha^{-2}}$ consisting of kinetic energy $W_k = \frac{m_e c^2}{2\alpha^{-2}}$ and potential energy $W_p = -\frac{m_e c^2}{\alpha^{-2}}$ expressed as[3]:

$$W_t = W_k + W_p = \frac{m_e c^2}{2\alpha^{-2}} - \frac{m_e c^2}{\alpha^{-2}} = -\frac{m_e c^2}{2\alpha^{-2}}. \tag{1}$$

Here m_e, α^{-1} and c is ground mass of electron, inverse fine structure constant and speed of light, respectively.

3. DOUBLE SURFACE

Double surface is average elliptic-hyperbolic sphere explaining the proposed heterogeneous curvature of the present world.[2] The infinite radius of the elliptic side of sphere enables uniform motion of the electron on the atom orbit.[4] Besides the orbit on elliptic sphere measuring the integer value of Compton wavelengths of the electron makes that motion stable. But at uniform motion the electron becoming free of centripetal force can leave Bohr orbit and land in other ground state orbits without any change in total energy. It just transforms the kinetic energy to potential energy and vice versa. The orbit on hyperbolic sphere measuring the non-integer value of Compton wavelengths of the electron[2] encourages the leaving.

4. THE NAME OF THE GROUND STATE ORBIT n

The length of the ground state orbit on the elliptic side of sphere (elliptic orbit n) is at the same time the name of the ground state orbit n. For instance Bohr orbit with $n = 137$ is named the 137^{th} orbit.

5. THE LENGTH OF ELLIPTIC ORBIT n

As already mentioned the elliptic orbit n should be for the sake of stability of integer value. The shortest length $n = 1$ still retains the wave nature of the electron. The longest length $n = 274$ is reached with the help of the maximal possible transformation of Bohr kinetic energy to potential energy. The average elliptic orbit $n = 137$ takes place on Bohr orbit.

6. THE LENGTH OF HYPERBOLIC ORBIT h

The length of the ground state orbit on the hyperbolic side of sphere (hyperbolic orbit h) is expressed by the elliptic orbit n as [2]:

$$h = n\left(2\sqrt{1+\frac{\pi^2}{n^2}}-1\right). \tag{2}$$

It is irrational. Since h yields the irrational value for all integer values of n due to the irrational value of π. Bohr orbit, for instance, has hyperbolic ground state orbit $h_{137} = 137.036072\ldots$[2]

7. THE LENGTH OF THE AVERAGE ELLIPTIC-HYPERBOLIC ORBIT s

The length of the average elliptic-hyperbolic orbit (elliptic-hyperbolic orbit s) is expressed by the elliptic orbit n as[2]:

$$s = n\left(2-\frac{1}{\sqrt{1+\frac{\pi^2}{n^2}}}\right). \tag{3}$$

It is irrational, too. Since s yields the irrational value for all integer values of n due to irrational value of π. Bohr orbit, for instance, has the elliptic-hyperbolic orbit $s_{137} = \alpha^{-1}$, the inverse fine structure constant.

8. CLIMBING UP AND DOWN FROM BOHR ORBIT

The electron can climb up from Bohr orbit to land on other ground state orbits in a way to gradually transform the kinetic energy $W_k = \frac{m_e c^2}{2\alpha^{-2}}$ to potential energy W_p . And vice versa, the electron can climb down from Bohr orbit to land on other ground state orbits gradually transforming potential energy $W_p = -\frac{m_e c^2}{\alpha^{-2}}$ to kinetic energy W_k. It can be examined that keeping the total energy of electron $W_t = -\frac{m_e c^2}{2\alpha^{-2}}$ untouched the relation between the elliptic-hyperbolic orbit $s = 2\pi r$ and corresponding kinetic energy $W_k(s)$ of the electron in the ground state of Hydrogen atom is the next:

$$W_k(s) = \frac{m_e c^2}{2\alpha^{-2}}\left(2\,x\,\frac{\alpha^{-1}}{s}-1\right). \tag{4}$$

Further, applying the relations (3) and (4) can be found out that the values of possible kinetic energies $W_k(s)$ on the ground state orbits are discrete and in the ground state of Hydrogen atom limited inside the next interval of elliptic orbits n:

$$1 \le n \le 2\,x\,137,\ \ n \in \mathbb{N}. \tag{5}$$

The number of orbits in the ground state of Hydrogen atom is thus 274.

9. CLIMBING UP TO THE MINIMAL KINETIC ENERGY

Climbing up the atom radius increases the elliptic-hyperbolic orbit $s = 2\pi r$. The rise is limited by the amount of kinetic energy of the electron on Bohr orbit $W_k(\alpha^{-1}) = \frac{m_e c^2}{2\alpha^{-2}}$ (1). The concerned kinetic energy would be exhausted $W_k(s) = 0$ on the orbit twice as large as Bohr orbit $s = 2\,x\,\alpha^{-1}$ so the maximal elliptic-hyperbolic orbit $s_{maximal}$ should not exceed that value:

$$s_{maximal} \le 2\,x\,\alpha^{-1}. \tag{6}$$

Consequently (3) the maximal elliptic orbit is the next:

$$n_{maximal} = 2\,x\,137 = 274. \tag{7}$$

And the maximal elliptic-hyperbolic orbit is the next:

$$s_{maximal} = 274.018008\ldots < 2\,x\,\alpha^{-1}. \tag{8}$$

According to the relation (4) on the maximal - 274^{th}- ground state orbit the electron possesses the minimal but non-zero kinetic energy:

$$W_{k,minimal} \approx 2.0\ x\ 10^{-4}\ x\ \frac{m_e c^2}{2\alpha^{-2}}. \tag{9}$$

10. CLIMBING DOWN TO THE MAXIMAL KINETIC ENERGY

Climbing down the atom radius decreases the elliptic-hyperbolic orbit $s = 2\pi r$. The downhill is not limited to the huge potential energy $W_p(Bohr) = -\frac{m_e c^2}{\alpha^{-2}}$ but to the length of the shortest - 1^{st}- ground state orbit. Consequently (3) the minimal elliptic-hyperbolic orbit is the next:

$$s_{minimal} = 1.696685\ldots \tag{10}$$

According to the relation (4) on the 1^{th} ground state orbit the electron possesses the maximal kinetic energy:

$$W_{k,maximal} \approx 160.5\ x\ \frac{m_e c^2}{2\alpha^{-2}}. \tag{11}$$

11. THE ELECTRON KINETIC ENERGY AND ELECTRON MASS

Applying the equations (3) and (4) the electron kinetic energy on the arbitrary n^{th} ground state orbit of Hydrogen atom is given by:

$$W_k(n) = \frac{m_e c^2}{2\alpha^{-2}}\left(\frac{2\alpha^{-1}}{n\left(2 - \frac{1}{\sqrt{1+\frac{\pi^2}{n^2}}}\right)} - 1\right), \qquad 1 \le n \le 274,\ \ n \in \mathbb{N}. \tag{12}$$

274 different kinetic energies of the electron in the range from $2.0\ x\ 10^{-4}\ x\ \frac{m_e c^2}{2\alpha^{-2}}$ to $160.5\ x\ \frac{m_e c^2}{2\alpha^{-2}}$ are possible. They contribute to the electron mass $m(n) = m_e(1 + \frac{W_k(n)}{c^2})$ $[5]$ so 274 different electron masses are possible, too.

12. THE ELECTRON SPEEDS

Mass m and speed a of the electron are related by panta rei equation$[5]$:

$$m^2c^2a^2 = e^{\frac{{m_e}^2c^2 - k(1-lnk) + m^2c^2(a^2-1)}{k}}. \tag{13}$$

Two speeds a belong to the non-ground mass m of the electron on every ground state orbit n. So, 274 pairs of speed are given solving panta rei equation for all 274 cases. The speeds in the next interval are given:

The lower speed:

$$0.0359667 < a_{lower} < 0.09163522. \tag{14}$$

The higher speed:

$$0.1626740 > a_{higher} > 0.0917801. \tag{15}$$

And the average speed $\bar{a}$ is extended in the next interval:

$$0.099320 > \bar{a} > 0{,}0917077. \tag{16}$$

13. THE ELECTRON POSITION RADII

The electron position radius Δr on some ground state orbit n is related to the difference of the electron speed Δa on that orbit.$[5]$ The relative change of the former mirrors the relative change of the later:

$$\frac{\Delta r}{\bar{r}} = \frac{\Delta a}{\bar{a}}. \tag{17}$$

It can be examined $(12), (16), (20)$ that the electron position radius is the greatest on Bohr orbit $n = 137$ and decreases against both extreme ground state orbits $n = 1$ and $n = 274$ as follows:

$$\Delta r_1 = 0{,}3444964\,\lambda_e < \Delta r_{137} = 2{,}4526011\,\lambda_e > \Delta r_{274} = 0{,}0688966\,\lambda_e. \tag{18}$$

And the sum of all ground state orbit position radii is the next:

$$\sum_{n=1}^{n=274} \Delta r_n = 527{,}9499 \ldots \lambda_e. \tag{19}$$

14. THE PROBABILITY OF FINDING THE ELECTRON ON GROUND STATE ORBITS OF HYDROGEN

The probability $P(n)$ of finding the electron on ground state orbit n depends on the time Δt_n spent on the same orbit. Let us propose that the electron prefers spatial comfort and spends more time Δt_n in the orbits n possessing greater position radius Δr_n than those possessing smaller one as follows:

$$P(n) = \frac{\Delta t_n}{\sum_{n=1}^{n=274} \Delta t_n} = \frac{\Delta r_n}{\sum_{n=1}^{n=274} \Delta r_n}. \tag{20}$$

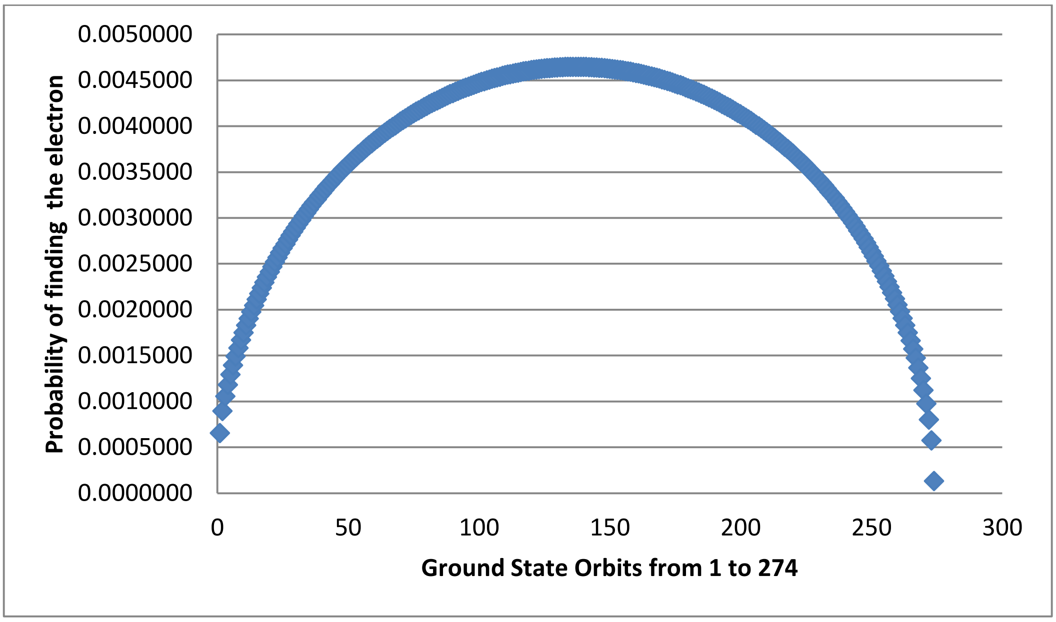
$$\sum_{n=1}^{n=274} P(n) = 1. \tag{21}$$

Applying the equations $(16), (20), (23)$ the probability $P(n)$ of finding the electron on the arbitrary ground state orbit n (Bohr orbit as well as finite number of orbits around it) could be found. The concerned probabilities are listed in the $Table 1$.

Table1. *Probability $P(n)$ of Finding Electron on Hydrogen Ground State Orbit n*

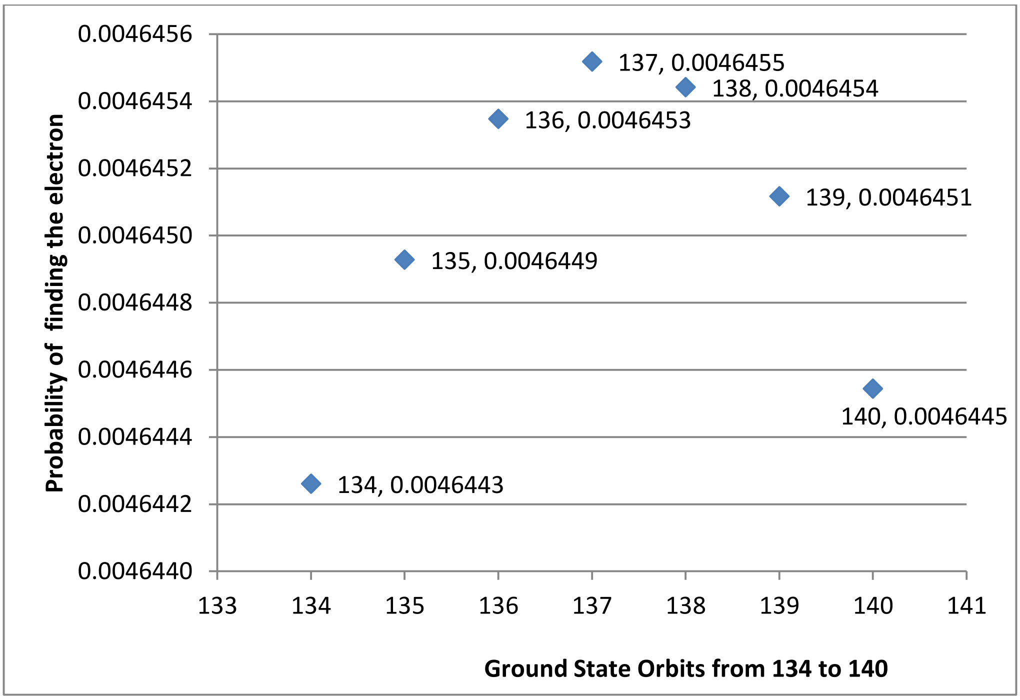
n	P(n)												
1	0,0006525	41	0,0033071	81	0,0042362	121	0,0046129	161	0,0045747	201	0,0041098	241	0,0030275
2	0,0008959	42	0,0033402	82	0,0042513	122	0,0046168	162	0,0045686	202	0,0040917	242	0,0029875
3	0,0010534	43	0,0033725	83	0,0042661	123	0,0046205	163	0,0045622	203	0,0040733	243	0,0029466
4	0,0011799	44	0,0034043	84	0,0042806	124	0,0046239	164	0,0045555	204	0,0040545	244	0,0029048
5	0,0012918	45	0,0034354	85	0,0042947	125	0,0046271	165	0,0045486	205	0,0040353	245	0,0028619
6	0,0013948	46	0,0034660	86	0,0043085	126	0,0046300	166	0,0045415	206	0,0040158	246	0,0028180
7	0,0014912	47	0,0034959	87	0,0043221	127	0,0046326	167	0,0045340	207	0,0039958	247	0,0027729
8	0,0015821	48	0,0035252	88	0,0043353	128	0,0046350	168	0,0045263	208	0,0039755	248	0,0027267
9	0,0016683	49	0,0035540	89	0,0043482	129	0,0046372	169	0,0045183	209	0,0039548	249	0,0026792
10	0,0017503	50	0,0035823	90	0,0043608	130	0,0046391	170	0,0045101	210	0,0039337	250	0,0026305
11	0,0018286	51	0,0036100	91	0,0043731	131	0,0046408	171	0,0045016	211	0,0039121	251	0,0025804
12	0,0019035	52	0,0036372	92	0,0043851	132	0,0046422	172	0,0044928	212	0,0038902	252	0,0025288
13	0,0019753	53	0,0036638	93	0,0043968	133	0,0046433	173	0,0044837	213	0,0038678	253	0,0024757
14	0,0020443	54	0,0036900	94	0,0044083	134	0,0046443	174	0,0044744	214	0,0038450	254	0,0024210
15	0,0021108	55	0,0037157	95	0,0044194	135	0,0046449	175	0,0044648	215	0,0038218	255	0,0023645
16	0,0021748	56	0,0037409	96	0,0044302	136	0,0046453	176	0,0044549	216	0,0037981	256	0,0023062
17	0,0022367	57	0,0037656	97	0,0044408	137	0,0046455	177	0,0044447	217	0,0037740	257	0,0022458
18	0,0022965	58	0,0037899	98	0,0044511	138	0,0046454	178	0,0044342	218	0,0037494	258	0,0021832
19	0,0023544	59	0,0038137	99	0,0044611	139	0,0046451	179	0,0044235	219	0,0037243	259	0,0021182
20	0,0024105	60	0,0038370	100	0,0044708	140	0,0046445	180	0,0044125	220	0,0036988	260	0,0020506
21	0,0024650	61	0,0038599	101	0,0044802	141	0,0046437	181	0,0044011	221	0,0036727	261	0,0019801
22	0,0025178	62	0,0038824	102	0,0044894	142	0,0046427	182	0,0043895	222	0,0036462	262	0,0019064
23	0,0025692	63	0,0039045	103	0,0044983	143	0,0046413	183	0,0043776	223	0,0036191	263	0,0018291
24	0,0026192	64	0,0039261	104	0,0045069	144	0,0046398	184	0,0043654	224	0,0035915	264	0,0017477
25	0,0026679	65	0,0039474	105	0,0045153	145	0,0046380	185	0,0043529	225	0,0035634	265	0,0016616
26	0,0027153	66	0,0039682	106	0,0045233	146	0,0046359	186	0,0043401	226	0,0035348	266	0,0015701
27	0,0027615	67	0,0039887	107	0,0045311	147	0,0046336	187	0,0043270	227	0,0035055	267	0,0014722
28	0,0028065	68	0,0040087	108	0,0045387	148	0,0046310	188	0,0043136	228	0,0034757	268	0,0013664
29	0,0028505	69	0,0040284	109	0,0045460	149	0,0046282	189	0,0042999	229	0,0034453	269	0,0012508
30	0,0028934	70	0,0040477	110	0,0045530	150	0,0046251	190	0,0042858	230	0,0034143	270	0,0011223
31	0,0029353	71	0,0040666	111	0,0045597	151	0,0046218	191	0,0042715	231	0,0033827	271	0,0009759
32	0,0029763	72	0,0040851	112	0,0045662	152	0,0046183	192	0,0042568	232	0,0033504	272	0,0008018
33	0,0030163	73	0,0041033	113	0,0045724	153	0,0046144	193	0,0042418	233	0,0033175	273	0,0005754
34	0,0030554	74	0,0041211	114	0,0045784	154	0,0046104	194	0,0042265	234	0,0032839	**274**	**0,0001305**
35	0,0030937	75	0,0041386	115	0,0045841	155	0,0046060	195	0,0042108	235	0,0032495		
36	0,0031312	76	0,0041557	116	0,0045895	156	0,0046015	196	0,0041948	236	0,0032145		
37	0,0031679	77	0,0041725	117	0,0045947	157	0,0045966	197	0,0041785	237	0,0031787		
38	0,0032038	78	0,0041889	118	0,0045996	158	0,0045915	198	0,0041619	238	0,0031421		
39	0,0032389	79	0,0042050	119	0,0046043	159	0,0045862	199	0,0041449	239	0,0031048		
40	0,0032734	80	0,0042208	120	0,0046087	160	0,0045806	200	0,0041275	240	0,0030666		

They are also figured in the $Graph$ 1:



Graph1. *Probabilty P(n) of Finding Electron on Hydrogen Ground State Orbit n*

And the highest of them are presented in the *Graph* 2:



Graph2. *Maximal Probability P(137) of Finding Electron on Some Hydrogen Ground State Orbit*

15. Conclusions

Treatise on the choice offers some discrete probability numbers for describing physical events on atom level.

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With best respects and sincere gratitude to high school times 1967-1970 spent in Prva gimnazija Maribor, Slovenia

Dedication

This fragment is dedicated to Sveti Jurij (Saint George) to shield recently opened unit of Pharmacy Špringer in Rogašovci, Goričko, Slovenia.

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