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Structural Evolution in Radium Nuclei Using IBM Consistent-Q Hamiltonian with Coherent State

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Abstract: *Using the interacting boson model (sd-IBM1) Hamiltonian in Casimir and multipole forms and the approach of intrinsic coherent state formalism, the evolution of shapes from spherical vibrator U(5) to deformed axially rotor Su(3) along the even-even Ra isotopes are investigated. The expectation value of the IBM Hamiltonian in the intrinsic coherent state for each nucleus provides the potential energy surface (PES) as a function of deformation parameters β and γ. The PES's are systematically analysed and the critical points are identified . We find that the ^{218}Ra, ^{220}Ra are vibrational while ^{226,228,230}Ra are rotational, the two nuclei ^{222}Ra and ^{224}Ra are transitional and close to the critical point symmetry X(5) limit. Also the PES's are rewritten in terms of the essential two parameters r_1,r_2 and the locus of the critical points in the essential parameter r_2-r_1 space are given. The X(5) predictions and our IBM calculations for Ra isotopic chain reproduce the energy ratios and the quadrupole transition probabilities B(E2). The vibrational to an axially prolate rotational shape phase transition is shown to take place quite smoothly as a function of boson number in the considered Ra isotopic chain. Some selected excitation energies and B(E2) values are calculated by using the PHINT code and a simulated fitting search program to derive the optimal best IBM parameters.*

Keywords: *shape phase transition, IBM*

1. INTRODUCTION

The study of nuclear shape phase transitions [1-7] have gained much theoretical interest, since the discovery of the critical point symmetries E(5) [8] and X(5) [9]. Most of these works have concentrated on the shape transition from spherical to deformed prolate [10,11] and the shape phase transition from spherical to γ- unstable [12-14]. In these studies there have been used several approaches, the most powerful approaches are the geometric collective model (GCM) [15, 16] and the interacting boson model [17]. The algebraic IBM was designed to describe the collective quadrupole degrees of freedom in medium mass and heavy nuclei. The IBM Hamiltonian was written from the beginning in second quantization form in terms of the generators of the U(6) group, subtended by s and d bosons which carry angular momenta 0 and 2 respectively. The three possible phases that can occur in the sd IBM for nuclei were classified as U(5), Su(3) and O(6), geometrically corresponding to spherical vibrator, axial rotator and γ-unstable rotation respectively. It was shown that the critical points of the first order shape phase transition between U(5) and Su(3) and the second order shape phase transition between U(5) and O(6) hold the critical point symmetries X(5) and E(5) respectively. The full shape of the transitional region can be characterized in terms of the Casten triangle [18]. It was shown that [19] the shape phase diagram depends on two independent combiners of the parameters IBM Hamiltonian called the essential parameters r_1 and r_2 which can be used to classify the equilibrium configure. Observables that are often used to follow the evolution of shape transitions along the isotopic chains are for example ratios of excitation energies $R_{L,2} = E(L_1^+)/E(2_1^+)$ electromagnetic transition such as the reduced quadrupole transition probabilities $B(E2, 2_1^+ \to 0_1^+), B(E2, 0_2^+ \to 0_1^+)$ and $B(E2, 2_2^+ \to 0_1^+)/B(E2, 2_2^+ \to 2_1^+)$, two neutron separation energies, isomer shifts and isotope shifts.

The plan of the present work can be divided into two stages: the first stage is to make use the consistent Q formalism of the original version of the IBM with the intrinsic coherent state approach to study the shape transition between the U(5) limit corresponding to a vibrating nucleus to the Su(3) limit corresponding to an axial-symmetric rotating nucleus. The second stage is to applied our results to the even-even Ra isotopic chain. The outline of the paper is as follows: In section 2, we presented the IBM Hamiltonian used. Section 3, is devoted to construct the PES 's by calculating the expectation value of the proposed Hamiltonian on the coherent state, and their evolution when moving between the different dynamical symmetries are studied. The critical points in the shape transition are identified and also the two essential parameters are given in section 4. In section 5, the dynamical symmetries U(5),Su(3) and O(6) are studied. In section 6, the characteristic quantities identifying the shape phases in the IBM are presented. Numerical calculations are performed in section 7, for even-even Ra isotopic chain and the advantages of the present approach are discussed. Finally, some concluding remarks are also given.

2. IBM1 Hamiltonian with One and Two Body Terms

The consistent Q formalism (CQF) [20] provides a simple and convenient three parameter space for IBM that span the entire Casten triangle. We make use the U(5)-Su(3) transitional Hamiltonian composed of the linear Casimir operator of the limit U(5), the quadratic Casimir operator of the limit Su(3) and the quadratic Casimir operator of the subgroup O(3)

$$H[U(5)-Su(3)] = \varepsilon_d C_1[U(5)] + \delta C_2[Su(3)] + \gamma C_2[O(3)] \tag{1}$$

Here $C_n[G]$ is the n-rank Casimir operator of the lie group G with

$$C_1[U(5)] = \hat{n}_d \tag{2}$$

$$C_2[O(3)] = 2(\hat{L}\,.\,\hat{L}) \tag{3}$$

$$C_2[Su(3)] = \frac{3}{4}\left(\hat{Q}\,.\,\hat{Q}\right) + \frac{1}{2}(\hat{L}\,.\,\hat{L}) \tag{4}$$

Where $\hat{n}_d$, $\hat{L}$ and Q^x are the d-boson number operator , the angular momentum operator and the quadrupole operator respectively, defined as :

$$\hat{n}_d = \sum_\mu d_\mu^\dagger \ \tilde{d}_\mu \tag{5}$$

$$\hat{L} = \sqrt{10}\ [d^\dagger \times \tilde{d}]^{(1)} \tag{6}$$

$$\hat{Q} = [s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} + \chi\,[d^\dagger \times \tilde{d}]^{(2)} \tag{7}$$

Where $s^\dagger$ (s) and $d^\dagger$ (d) are monopole and quadrupole boson creation (annihilation) operators respectively and χ is the structure parameter and it is shown by microscopic theory to lie between $-\sqrt{7}/2$ and $\sqrt{7}/2$.

In terms of the multipole operators $\hat{n}_d$, $\hat{L}$ and $\hat{Q}$, the IBM Hamiltonian (1) can be rewritten as:

$$H[U(5)-Su(3)] = \varepsilon_d \hat{n}_d + a_1\,\hat{L}\,.\,\hat{L} + a_2\,\hat{Q}\,.\,\hat{Q} \tag{8}$$

Where $a_1 = 2\gamma + \frac{1}{2}\delta$, $a_2 = \frac{4}{3}\delta$ (9)

and we introduce the scalar products

$$A^{(l)}\,.\,B^{(l)} = \sqrt{2l+1}\,[A^{(l)} \times A^{(l)}]_0^{(0)} \tag{10}$$

$$\hat{L}\,.\,\hat{L} = -5\sqrt{3}\,[[d^\dagger \times \tilde{d}]^{(1)} \times [d^\dagger \times \tilde{d}]^{(2)}\ ]_0^{(0)} \tag{11}$$

$$\hat{Q}\,.\,\hat{Q} = \frac{1}{2}\sqrt{5}\,[([s^\dagger\,\tilde{d} + d^\dagger\,s]^{(2)} + x\,[d^\dagger \times \tilde{d}]^{(2)}) \times ([s^\dagger\,\tilde{d} + d^\dagger\,s]^{(2)} + x\,[d^\dagger \times \tilde{d}]^{(2)})\,]_0^{(0)} \tag{12}$$

The interaction parameters ε, a_1, a_2 in terms of code PHINT [21] notation EPS, ELL and QQ are

$$\varepsilon_d = EPS,\ a_1 = \frac{1}{2}ELL\ ,\ a_2 = QQ \tag{13}$$

3. The Classical Energy Limit of The Hamiltonian

In order to analyze the shape phase structure of our model, an intrinsic coherent state for the IBM Hamiltonian was proposed [17,22,23] in terms of shape parameters β and γ. In this approach, the ground state is a variational state built out of bosons defined by the creation operator

$$\Gamma^{\dagger} = \frac{1}{\sqrt{1+\beta^2}}\left[s^{\dagger} + \beta \cos\gamma \; d_0^{\dagger} + \frac{1}{\sqrt{2}}\beta \sin\gamma \left(d_2^{\dagger} + d_{-2}^{\dagger} \right)\right] \tag{14}$$

And the N boson condensate is

$$|N\beta\gamma\rangle = \frac{1}{\sqrt{N!}} (\Gamma^{\dagger})^N |0\rangle \tag{15}$$

where N is the total number of bosons and $|0\rangle$ is the boson vacuum. The intrinsic shape variables β and γ are the order parameters of the nucleus, the deformation parameter β measure the axial deviation from sphericity, while the angle variable γ controls the departure from axial symmetry. We set γ=0 to study only the β dependence.

The expectation value of the Hamiltonian (8) in the intrinsic coherent state (15) provides the potential energy surface (PES) of the nucleus. The PES in terms of the parameters of the Hamiltonian and deformation parameter β can be written as

$$E(N,\beta) = \langle N,\beta\,|H|\,N,\beta\rangle = c_1 \frac{N\beta^2}{1+\beta^2} + \frac{N(N-1)}{(1+\beta^2)^2}[c_2\beta^2 + c_3\beta^3 + c_4\beta^4] + c_0 \tag{16}$$

where $c_1 = \sigma + (x^2 - 4)\, a_2$ (17)

$c_2 = 4\, a_2$ (18)

$c_3 = -4\sqrt{2/7}\, x \;\; a_2$ (19)

$c_4 = \frac{2}{7} x^2 a_2$ (20)

$c_0 = 5\, N\, a_2$ (21)

with $\sigma = \varepsilon_d + 6\, a_1$ (22)

The shape of the nucleus is defined through the equilibrium value of the deformation parameter β which is obtained by minimizing the ground state energy E(N, β). A spherical nucleus has a global minimum on PES at β=0 where as a deformed one has minimum at $\beta \neq 0$.

If we introduce coctrol parameter λ such that

$$\lambda = \frac{-a_2}{\sigma}(N-1) \tag{23}$$

Then the PES (16) depends only on two parameters λ and χ and take the form

$$\varepsilon(N,\beta) = \frac{E(N,\beta)}{\sigma} = (1 - e\lambda)\frac{N\beta^2}{1+\beta^2} + \frac{N\lambda}{(1+\beta^2)^2}\left[-4\beta^2 + 4\sqrt{\frac{2}{7}}\, x\beta^3 - \frac{2}{7}x^2\beta^4\right]$$

$$-5\lambda\frac{N}{N-1} = \frac{A_2\beta^2 + A_3\beta^3 + A_4\beta^4}{(1+\beta^2)^2} + A_0 \tag{24}$$

where $A_2 = [1 - (4 + e\lambda)]N$ (25)

$A_3 = 4\sqrt{2/7}\, x\lambda N$ (26)

$A_4 = [1 - \left(\frac{2}{7}x^2 + e\right)\lambda]N$ (27)

$A_0 = -5\lambda\frac{N}{N-1}$ (28)

with $e = \frac{x^2-4}{N-1}$ (29)

4. Critical Behavior in U(5)-Su(3) Shape Phase Transition

To analize the critical behavior for the energy functional equation (24). The antispinodal point occur when $\varepsilon(N,\beta)$ becomes flat at β =0 or when $\left.\frac{\partial^2\varepsilon}{\partial\beta^2}\right|_{\beta=0} = 0$, ($A_2$=0), the critical point occur when ${A_3}^2 = 4A_2A_4$ and the equilibrium value of β occur when the first order derivative of $\varepsilon(N,\beta)$ with respect to β vanish $\frac{\partial\varepsilon}{\partial\beta} = 0$ These conditions yield the following for antispinodal point

$$\lambda_a = \frac{1}{4+e} \tag{30}$$

for critical point

$$1-\left(4+\tfrac{2}{7}x^2+2e\right)\lambda_c+\left(\tfrac{2}{7}x^2+4\right)e\lambda_c^2=0 \quad (31)$$

for equilibrium yield the cubic equation

$$2A_2+3A_3\beta_e+(4A_4-2A_2)\beta_e^2-A_3\beta_e^3=0 \quad (32)$$

According to the catastrophe theory [19,24,25] the PES can be rewritten in a special form in terms of two essential parameters r_1 and r_2 which gives the locus of the critical points in plane forming by r_2, r_1. The essential parameters r_1 and r_2 are defined as:

$$r_1=\frac{c_2+\frac{c_1}{N-1}}{2c_4+\frac{c_1}{N-1}-c_2}\ ,\qquad r_2=\frac{-2c_3}{2c_4+\frac{c_1}{N-1}-c_2} \quad (33)$$

In terms of the control parameter λ the essential parameters takes the form,

$$r_1=\frac{1-\lambda(4+\mathrm{e})}{1-\lambda(\frac{4}{7}x^2-4+\mathrm{e})} \quad (34)$$

$$r_2=\frac{-8\sqrt{2/7}\,x\,\lambda}{1-\lambda(\frac{4}{7}x^2-4+\mathrm{e})} \quad (35)$$

For axially symmetric deformed prolate rotator $x=-\sqrt{7/2}$, then the coefficients A_2, A_3 and A_4 of equation (24) becomes

$$A_2=\left[1-\left(4-\frac{9}{4N-4}\right)\lambda\right]\mathrm{N} \quad (36)$$

$$A_3=-2\sqrt{2}\,\lambda\,\mathrm{N} \quad (37)$$

$$A_4=\left[1-\left(\frac{1}{2}-\frac{9}{4N-4}\right)\lambda\right]\mathrm{N} \quad (38)$$

The corresponding antispinodal λ_a and critical λ_c points are

$$\lambda_a=\frac{4N-4}{16N-25} \quad (39)$$

$$1-\left(\frac{9}{2}+2e\right)\lambda_c+\frac{9}{2}e\,\lambda_c^2=0 \quad (40)$$

with $e=\frac{-9}{4N-4}$ (41)

If we eliminate the contribution of one-body terms of the quadrupole-quadrupole interaction (e = 0), the PES takes the form (when $x=-\sqrt{7/2}$)

$$\varepsilon(N,\beta)=\frac{\mathrm{N}\beta^2}{1+\beta^2}-\frac{\mathrm{N}\lambda}{(1+\beta^2)^2}\left[4\beta^2+2\sqrt{2}\,\beta^3+\frac{1}{2}\beta^4\right]$$
$$=\mathrm{N}\frac{(1-4\lambda)\beta^2-2\sqrt{2}\beta^3+(1-\frac{1}{2}\lambda)\beta^4}{(1+\beta^2)^2} \quad (42)$$

In this case the antispinodal and critical points are located at

$$\lambda_a=\tfrac{1}{4}\ \ ,\lambda_c=\tfrac{2}{9} \quad (43)$$

The corresponding essential parameters r_1 and r_2 of the shape diagram are

$$r_1=\frac{1-4\lambda}{1+3\lambda}\ \ ,\ \ r_2=\frac{4\sqrt{2}\,\lambda}{1+3\lambda} \quad (44)$$

Therefore

$$r_{1a} = 0 \quad , \quad r_{2a} = \frac{4\sqrt{2}}{7} \tag{45}$$

$$r_{1c} = \frac{1}{15} \quad , \quad r_{2c} = \frac{8\sqrt{2}}{15} \tag{46}$$

The equilibrium value of the deformation parameter β is given by solving the cubic equation

$$(1-4\lambda) - 3\sqrt{2}\,\lambda\,\beta_e + (1+3\lambda)\beta_e^{\,2} + \sqrt{2}\,\lambda\,\beta_e^{\,3} = 0 \tag{47}$$

The deformation parameter β at the critical point (λ_c =2/9) is given by

$$\beta_e = \frac{1}{2\sqrt{2}} = \frac{r_{2c}}{1 + \sqrt{1 + \frac{1}{2} r_{2c}^{\,2}}} \tag{48}$$

To illustrate the critical behavior in the shape phase transition, a sketch of the U(5)-Su(3) evolution for $x = -\sqrt{7}/2$ and large N limit is shown in Figure (1) for $\lambda_c \leq \lambda \leq \lambda_c$. For $\lambda = 1/9$, the nucleus is in the symmetric phase since the PES has a unique minimum at $\beta = 0$. When λ increase to critical point $\lambda = 9/11$, the nonsymmetric and symmetric minima attain the same depth, greater than this value, the symmetric minimum at $\beta = 0$ becomes a local minimum till $\lambda = 1/4$ where it becomes unstable antispinodal point.

5. Dynamical Symmetry Limits U(5) and Su(3)

(i) For $a_1=a_2=0$, the original Hamiltonian of equation (8) reduces to the vibrational U(5) limit of the IBM

$$H[U(5)] = \varepsilon_d \hat{n}_d \tag{49}$$

The corresponding PES is given by

$$E[U(5)] = \varepsilon_d \frac{N\beta^2}{1+\beta^2} \tag{50}$$

In this limit the essential parameters r_1, r_2 and the equilibrium value of β are given by $r_1=1$, $r_2=0$, $\beta_e = 0$

(ii) For $\varepsilon_d = a_1 = 0$ and $x = -\frac{\sqrt{7}}{2}$, $\gamma = 0$ the Hamiltonian (8) reduces to the rotational Su(3) limit (axially deformed shape)

$$H[Su(3)] = a_2\,\hat{Q}(x = -\sqrt{7}/2)\,.\,\hat{Q}(x = -\sqrt{7}/2) \tag{51}$$

And if we eliminate the contribution of the one body terms of the quadrupole-quadrupole interaction, the PES reads

$$E[Su(3)] = a_2 \frac{N(N-1)}{(1+\beta^2)^2}\left[4\beta^2 + 2\sqrt{2}\,\beta^3 + \frac{1}{2}\beta^4\right] \tag{52}$$

The essential parameters r_1,r_2 and the equilibrium value of β are given by

$$r_1 = -\frac{4}{3} \quad , \quad r_2 = \frac{-4\sqrt{2}}{3} \, , \quad \beta_e = \sqrt{2} \tag{53}$$

6. Other Tests of the Critical Point Behavior

In several U(5)-Su(3) transitional nuclei, the low lying energy ratios $R_{L,2}$ and the ratios of the electric quadrupole reduced transition probabilities $B_{L,2}$ reproduce the X(5) critical point symmetry.

For ground state band the energy ratio $R_{L,2}$ is defined as

$$R_{L,2} = \frac{E(L_1^+)}{E(2_1^+)} = \begin{cases} \dfrac{L}{2} & for\ U(5) \\ \dfrac{L(L+1)}{6} & for\ Su(3) \end{cases} \tag{54}$$

The electric quadrupole reduced transition probabilities $B_{L,2}$ is defined as

$$B(E2, L_i \rightarrow L_f) = \frac{1}{2L_i + 1} \left| \langle L_f \| T(E2) \| L_i \rangle \right|^2 \tag{55}$$

where L_i and L_f are angular momenta of the initial and final states respectively. The E2 transition operator T(E2) is given by

$$T(E2) = e\,\hat{Q} \tag{56}$$

with e being the boson effective charge. The ratios $B_{L,2}$ for the U(5) and Su(3) dynamical symmetry limits are given by

$$B_{L+2,2} = \frac{B(E2, L+2 \rightarrow L)}{B(E2, 2_1^+ \rightarrow 0_1^+)} = \begin{cases} \frac{1}{2}(L+2)(1-\frac{L}{2N}) & for\ U(5) \\ \frac{15}{2}\frac{(L+2)(L+1)}{(2L+3)(2L+5)}(1-\frac{L}{2N})(1+\frac{L}{2N+3}) & for\ Su(3) \end{cases} \tag{57}$$

7. Numerical Results

(i) Derived IBM parameters

The χ^2 test is used in the fitting produce in order to extract the optimal best parameters of the IBM Hamiltonian. The χ^2 function is defined in the standard way as

$$\chi^2 = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \left| \frac{x_i(data) - x_i(IBM)}{\Delta\, x_i(data)} \right|^2$$

where N_{data} is the number of experimental date, $x_i(data)$ describe the experimental excitation energy of some selected energy levels and some selected B(E2) values, and $x_i(IBM)$ denotes the corresponding calculated IBM values and $\Delta\, x_i(data)$ assigned the experimental errors to each $x_i(data)$ point. The minimization is carried out for each isotope separately using PHINT and a simulated fitting search program to derive the optimal best IBM parameters.

(ii) Evolution of low-lying spectra

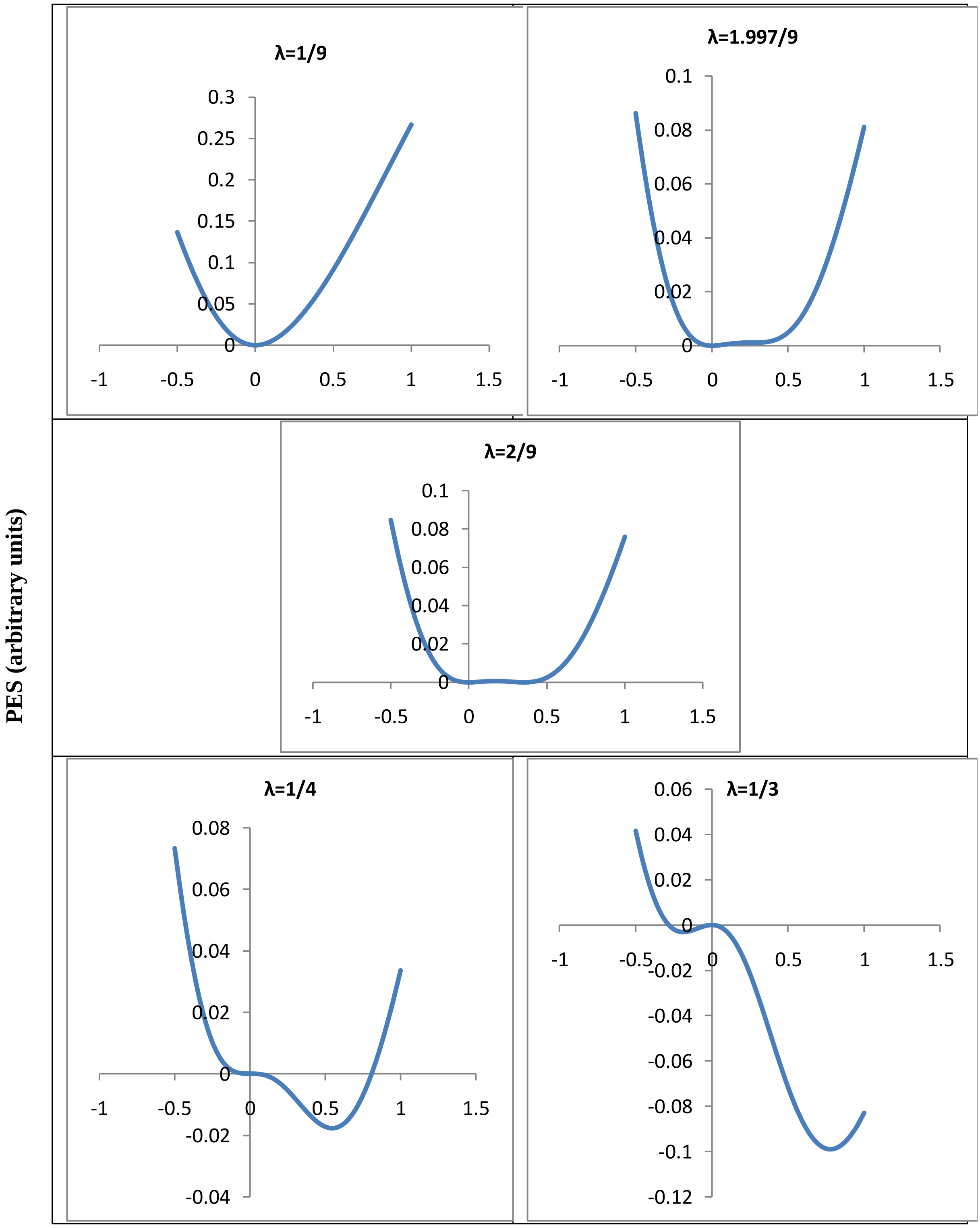
A phase transition in nuclear shape exhibit a sharp change in the excitation energy of the first 2_1^+ level and energy ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$ as a function of the total boson number N_B along the considered isotopic chain. Figure (2) illustrate $E(2_1^+)$ and R_{42} as a function of N_B for the Ra isotopic chain. The decrease of $E(2_1^+)$ with increasing N_B shows a corresponding increase of collectivity and we observe a transition between vibrational $R_{4/2}$ =2 (for lighter isotopes) to clear rotational $R_{4/2}$ =3.33 for heavier isotopes above A=226 with critical point located at ^{224,226}Ra (N_B =7,8). Figure (3) shows the energy ratios R_{L2} for Ra isotopic chain compared to the U(5), X(5) are Su(3) symmetry limits. We see that ^{118,120}Ra are near the U(5) (vibrational) while ^{226,228,230}Ra are near Su(3)(rotational), the two nuclei ^{222,224}Ra are close to the critical point symmetry X(5) limit. In figure (4) the calculated $B_{L+2,2}$ ratios for the best candidate ^{224}Ra (N_B=8 which is close to the critical point symmetry X(5)) are compared to the U(5) and Su(3) predictions.

The PES 's $E(N,\beta)$ with the coefficients listed in Table (1) according to $x = -\frac{\sqrt{7}}{2}$, $\gamma = 0$ are shown in Figure (5) for Ra isotopic chain. From the graph we observe the evolution from a spherical potential (N=5, 6), where minimum is found at $\beta = 0$ to potentials with well deformed minma $(N > 8)$. For intermediate (N=7, 8) the surfaces display the typical flat. bottomed curve expected at the critical points.

An analysis with catastrophe theory shows that the values of the essential parameters r_1,r_2 for Ra isotopic chain which exhibit a transitional region between the U(5) and Su(3) is characterized by a straight line illustrated in Figure (6). The numerical values of r_1, r_2 are listed in Table (1). The dotted line in Figure (6) join the two pure dynamical symmetry limits U(5) and Su(3).

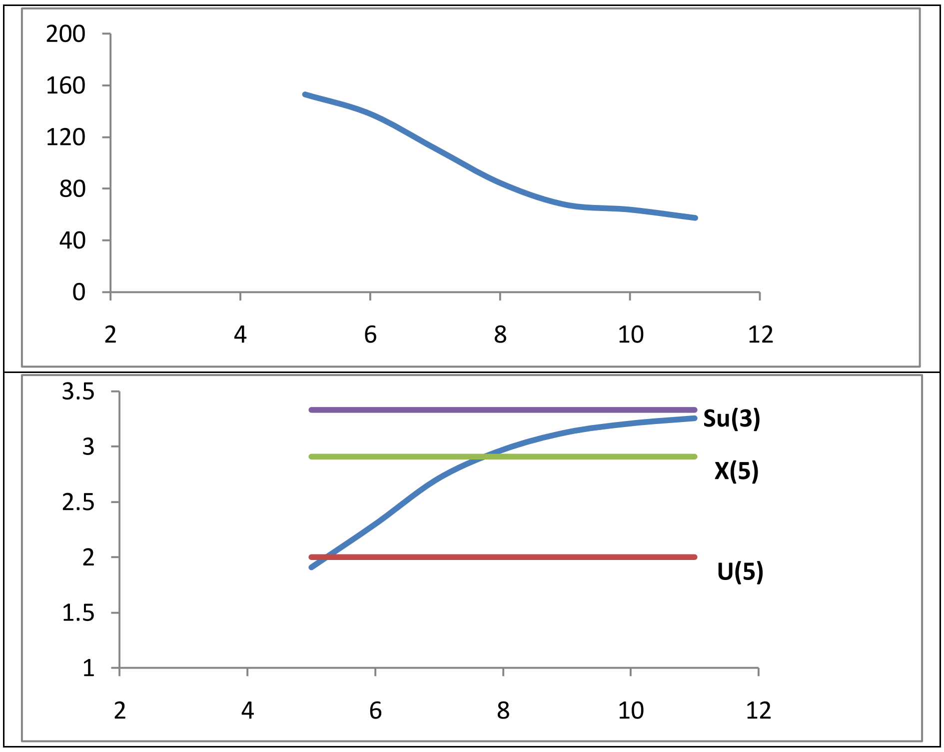
Table1. *The optimized fitted parameters A_2, A_3, A_4 ,A_0 (in KeV) for Ra isotopic chain. N_B is the total number of bosons and r_1, r_2 are the essential parameters.*

Isotope	N_B	A_2	A_3	A_4	A_0	r_2	r_1
^{218}Ra	5	3130.9487	-970.2070	4077.2872	-10.0100	0.38625	0.65907
^{220}Ra	6	2400.7965	-1455.3106	3820.3044	113.6175	0.55548	0.50971
^{222}Ra	7	1177.2302	-2037.4348	3164.5412	279.1215	0.79095	0.30187
^{224}Ra	8	436.1500	-2716.5490	3085.8980	486.502	0.94726	0.16391
^{226}Ra	9	-1263.0447	-3492.7454	2143.7746	735.759	1.25851	-0.11056
^{228}Ra	10	-3594.9525	-4365.9318	663.5711	1026.8925	1.77401	-0.56571
^{230}Ra	11	-4972.5748	-5336.1388	232.2874	1359.9025	1.96284	-0.73247



Deformation parameter (β)

Figure (1) PES for U(5)-Su(3) transition in the large N limit and ignoring the contribution of the body terms of quadrupole-quadrupole interaction for different λ values as a function of the deformation parameter β.



Boson Number (N_B)

Figure2. *Evalution of excitation energy of the first 2⁺ level and energy ratio $E(4_1^+)/E(2_1^+)$ as a function of the boson number N_B in the isotopic chain of Ra.*

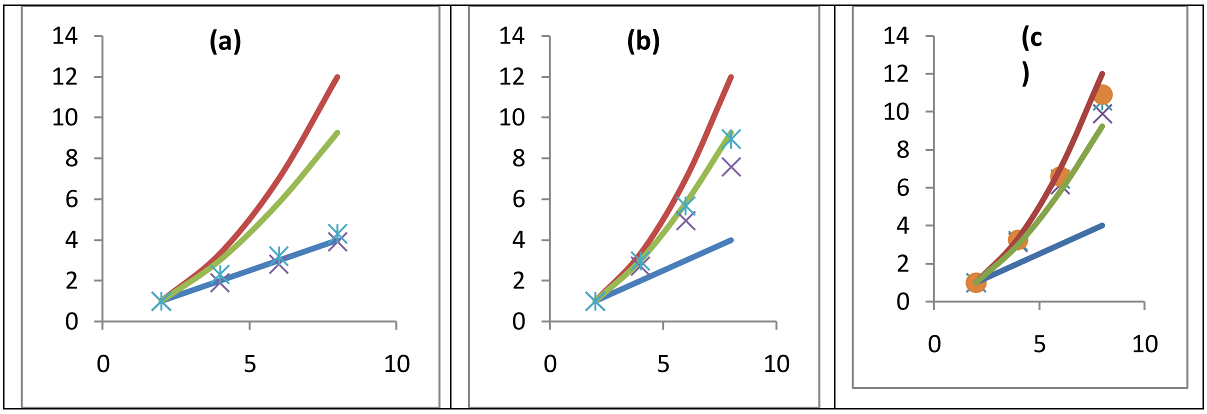


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Figure3. *Comparison of the ratios $R_{L,2} = E(L_1^+)/E(2_1^+)$ of the ground state bands in U(5), X(5),Su(3) prediction with the IBM calculations for (a) ^{218,220}Ra (b)^{222,224}Ra and (c) ^{226,228,230}Ra*

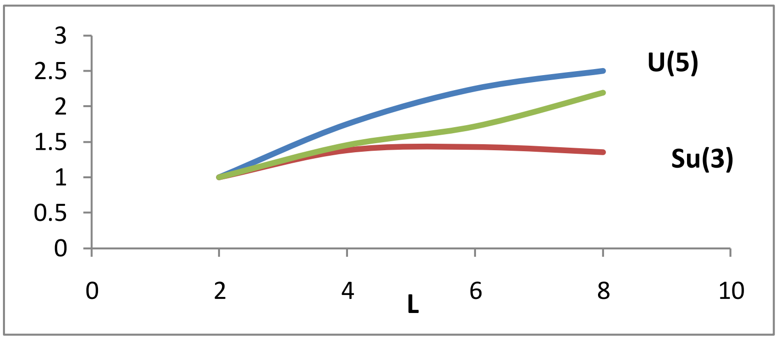

Figure4. *Comparison of the $B_{I+2,2}$ ratios of the ground state band in ^{224}Ra (N_B=8) compared to the U(5) and Su(3) predictions.*

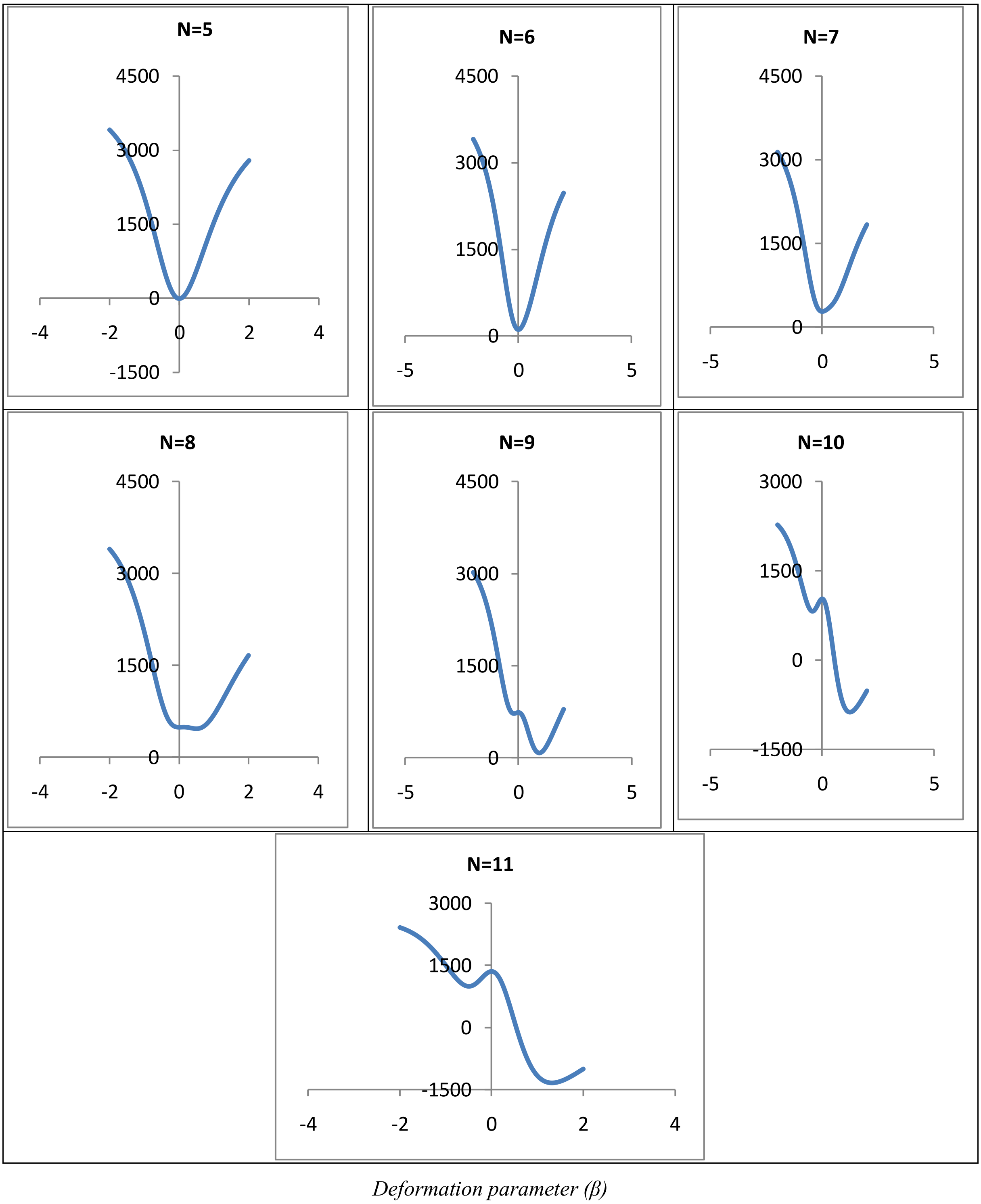

Figure5. *PES 's E(β) in terms of the deformation parameter for Ra isotopic chain with boson number varying from N=5 to N=11 to describe the IBM U(5)-Su(3) shape phase transition.*

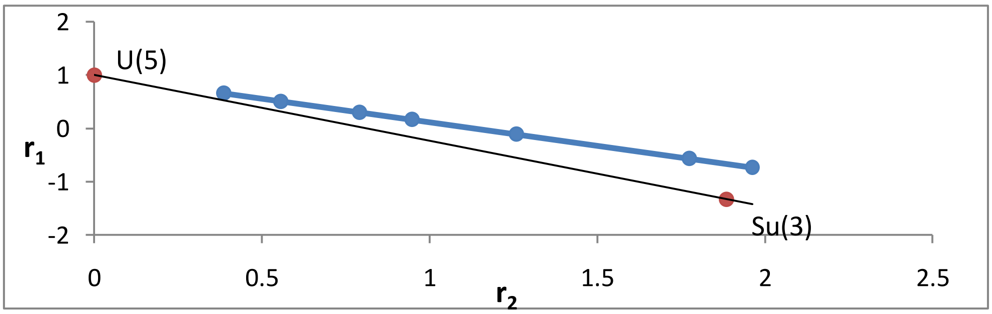

Figure6. *Shape phase diagram for Ra isotopes in terms of the essential parameters r_1 and r_2.*

8. Concluding Remarks

The transition from the spherical U(5) dynamical symmetry to axially deformed prolate rotor Su(3) dynamical symmetry has been studied. The Hamiltonian describing this transition is the consistent-Q interacting boson model Hamiltonian depending on a control parameter λ and structure parameter χ , that leads to the potential energy surface (PES) by using the intrinsic coherent state formalism. We have analysed the critical points of the shape phase transitional region U(5)-Su(3) in the space of the control parameter λ by the variation of boson number. The essential parameters r_1 and r_2 have been extracted in terms of the coefficients appearing in the PES and used to classify the equilibrium configurations. The IBM calculations and the symmetry limits are represented in the r_2-r_1 plane. Our model has been applied to the even-even Ra isotopic chain. For each isotope a general fit is performed to get the PES coefficients. As a result, we find that ^{222}Ra and ^{224}Ra are the best candidates to be critical and close to the critical point symmetry X(5). To identify the shape phase and their transitions we examined the fluctuation in the energy ratios and the ratios of the E2 transition rates.

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