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Exploring Intruder Levels of Nuclei (^{96}Zr, ^{98}Zr, ^{98}MO) Within the Framework of IBM-2 Model

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Abstract: *Current work includes a study of the very rare case called intruder nuclear levels, where there are only seven nuclei in nature. Such cases occur when the first excited state is* 0_2^+*. The current study included only three nuclei:* $^{98}_{42}Mo$*,* $^{98}_{40}Z$ *and* $^{96}_{40}Zr$*. The nuclear model used to explore and investigate nuclei in this work is the second interacting boson. The experimental data and theoretical values obtained were in good agreement, indicating this model's success in calculating such anomalies. The main reason for the presence of these levels in other than their natural locations expected according to the IBM model is that they have a double subshell closure. The majority of the theoretical values computed by IBM-2 aligned well with the experimental values for energy levels, decreased transition probabilities, electric quadrupole transitions, magnetic dipole transitions, and zero transitions. This research aims to contribute to the knowledge of the nuclear characteristics of the isotopes currently under investigation.*

Keywords: *IBM-2, Energy levels, Reduced transition probability, Electric quadrupole moment, Intruder nuclear levels, Electric Monopole Transitions,*

1. INTRODUCTION

The theoretical examination of nuclear characteristics poses a significant challenge for physicists owing to the intricate nature of nuclear forces and nucleon interactions. Consequently, the majority of nuclear models relied on physical and mathematical approximations [1-5]. The most effective mathematical model to elucidate nuclear features so far is the interacting boson model introduced by the scientists (Iachello and Arima) in (1978), which successfully integrates concepts from the shell model and the collective model. Multiple iterations of this concept emerged, with IBM-2 being the most prevalent due to its advancement from IBM-1, which differentiated between the proton boson and the neutron boson [6-10]. A substantial amount of study was conducted owing to the precision of the outcomes in this. Nuclei are categorized into three categories depending on the form of the nucleus and its motion: spherical vibration, distorted rotation, and an intermediate variety known as soft. This categorization was included into the interacting boson model, derived from collective models, which has the Kasten triangle, with all nuclei in nature positioned along its sides. The nuclei in this study work are of the vibrational type, with their classifications established according to the ratio $E(4_1^+)/E(2_1^+)$ [11-14]. The distinguishing feature of these nuclei is the significant extremity in this ratio, which has a usual value of 2. The current nuclei values are 1.57 for $^{96}_{40}Zr$, 1.5 for $^{98}_{40}Z$, and 1.9 for $^{98}_{42}Mo$. The values are deemed minimal, indicating that the selected nuclei possess a pronounced vibrational character, resulting in a rare phenomenon in nature: the emergence of the second excited azimuth 2_2 as the first excited level following zero, the rationale for which will be elucidated in the discussion section. This phenomenon motivated our investigation of this sort of nuclei, since only seven nuclei in nature exhibit this peculiar condition, which is $\boldsymbol{^{98}_{42}Mo}$, $\boldsymbol{^{98}_{40}Z}$, $\boldsymbol{^{96}_{40}Zr}$, $\boldsymbol{^{90}_{40}Zr}$, $\boldsymbol{^{96}_{32}Ge}$, $\boldsymbol{^{40}_{20}Ca}$,, $\boldsymbol{^{16}_{8}O}$. Utilizing the equations of the IBM-2 model, we calculated these energy levels, which we designated as Intruder Levels. This demonstrates the model's capacity to compute this category of energy levels. We relied on the concepts of the shell model due of their proximity to reality. The aims that prompted this investigation may be succinctly expressed as follows:

1- Investigating the nuclear characteristics of these isotopes using the principles of the interacting boson model. Monitoring and analyzing an unusual phenomenon seen in nature in just seven nuclei, namely the emergence of level 2_2 as the initial excited state, and correlating the pronounced vibrational characteristics of these isotopes with this occurrence.

2- Investigating further features, including electrical and magnetic transitions, as well as zero transitions of these nuclei.

2. Hamiltonian Formulation

The essential mathematical construct in nuclear physics is the Hamiltonian, which encapsulates the total energy of a nuclear system. The IBM-2 Hamiltonian describes the interactions and energies of nucleons within the nucleus, incorporating both collective and intrinsic degrees of freedom. It is a crucial element in our investigation as it provides the foundation for predicting energy levels and transitions in Zirconium and Molybdenum nuclei [11-15].

Mathematically, the IBM-2 Hamiltonian can be expressed as [20]:

$$\hat{H} = \varepsilon_d(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa(\hat{Q}_\pi.\hat{Q}_\nu) + \sum_{\rho=\pi,\nu} \hat{V}_{\rho\rho} + \hat{M}_{\pi\nu}(\xi_1, \xi_2, \xi_3) \quad \dots\dots\dots\dots \quad (1)$$

where, ε_d is the single-particle energy of the d bosons, κ denotes the strength of the quadrupole-quadrupole interaction term, n_d is the number operator for d bosons, the quadruple operator $\hat{Q}_\rho$ is given by $Q_\rho = [d_\rho^+ \times s_\rho^\sim + s_\rho^+ \times d_\rho^+]^{(2)} + \chi_\rho[d_\rho^+ \times d_\rho^\sim]^{(2)}$, the mentioned symbol$\rho$ refers toπ orν[16], the last term $\hat{M}_{\pi\nu}$is called Majorana term depends on parameters $\xi_{1,2,3}$ the purpose of this term was to shit the energy state with mixed symmetry with respect to the totally symmetric one.

The interaction between like-bosons ($\hat{V}_{\rho\rho}$) are sometimes included to improve the fit to experimental energy spectra, has the following expression [17]:

$$\hat{V}_{\rho\rho} = \sum_{L=0,2,4} C_\rho^L \left([d_\rho^+ d_\rho^+]^{(L)}.[d_\rho^\sim d_\rho^\sim]^{(L)}\right) \quad \dots\dots\dots\dots \quad (2)$$

For simplicity's sake, one can halve the number of the free parametersC_ρ^L.

3. Electromagnetic Transition Equations

In addition to energy levels, we calculate electromagnetic transitions between nuclear states using the IBM-2 framework. The electromagnetic transition probability can be computed using the Wigner-Eckart theorem and can be expressed as[18]:

$$B(E\lambda; J_i^+ \to J_f^+) = \left|< J_f^+, M_f | T(E\lambda,\mu) J_i^+, M_i >\right|^2 \cdots\cdots\cdots \quad (3)$$

where $B(E\lambda; J_i^+ \to J_f^+)$ is the electromagnetic transition probability, λ denotes the multipolarity (electric or magnetic), and $T(E\lambda,\mu)$ is the transition operator.

The electric quadrupole transition operator T(E2) in the IBM-2 can be determined according to the reduced form equation[19]:

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \cdots\cdots\cdots \quad (4)$$

The dipole magnetic transition operator T(M1) is a combination of the angular momenta of protons and neutrons, $T^{(M1)} = \sqrt{3/4\pi}\,(g_\pi L_\pi + g_\nu L_\nu)$, and commonly written as the following [20]:

$$T^{(M1)} = 0.77\ [(d^+ \times d^\sim)_\pi^{(1)} - (d^+ \times d^\sim)_\nu^{(1)}](g_\pi - g_\nu) \cdots\cdots\cdots \quad (5)$$

The electromagnetic transitions of the nucleus are often not pure; for example, a level may be decayed by a quadrupole electromagnetic transition, as well as by a magnetic dipole, and this phenomenon occurs due to mixed symmetry (MSSs). In order to find out which of the two transitions was the dominant (strongest), it is necessary to know the so-called mixing ratio, which is written as follows [21]:

$$\delta\left(\frac{E2}{M1}; J55_i^+ \to J_f^+\right) = 0.835 E_\gamma[MeV] \times \frac{< J_f^+ \| T^{(E2)} \| J_i^+ >}{< J_f^+ \| T^{(M1)} \| J_i^+ >} \cdots\cdots\cdots \quad (6)$$

4. The Calculations and Comparison with Experimental Data

Our methodology follows established procedures in nuclear structure research, combining theoretical modeling with data-driven analysis to explore and elucidate the nuclear properties of Zr and Mo nuclei[22]. The relevant experimental data was taken from the *Nuclear National Data Center*; then special software based on the IBM-2 model (NPBOS & NPBTRN) was used to perform calculations [23]. The parameters of the Hamiltonian have been set to get the best fit of energy level values to the experimental values. Comparison between theory and experiment guides further refinements of our model parameters. The comparison with experimental data is essential for validating the accuracy of our calculations and for assessing the predictive power of the IBM-2 model [24].

5. Results and Discussion

The reason for choosing ^{96}Zr, ^{98}Zr, and ^{98}MO nuclei is that it is characterized by a phenomenon that is almost rare in nature, which is the appearance of energy level 0_2^+ as the first excited level after 0_1^+ directly. This unique phenomenon only appears in seven nuclei: $^{98}_{42}\boldsymbol{Mo}$, $^{98}_{40}\boldsymbol{Z}$, $^{96}_{40}\boldsymbol{Zr}$, $^{90}_{40}\boldsymbol{Zr}$, $^{96}_{32}\boldsymbol{Ge}$, $^{40}_{20}\boldsymbol{Ca}$,, $^{16}_{8}\boldsymbol{O}$. The appearance of this energy level is considered a strange and rare case because it is known that in most even-even nuclei, the first excited level is 2_1^+. To discuss the energy levels of these nuclei, we first determined the affiliation of the nuclei. As is known, nuclei in nature are classified into three main groups: deformed rotational nuclei whose group symbol is SU(3), vibrational nuclei U(5), and transitional nuclei O(6). According to the collective models, these nuclei belong to the group of vibrational nuclei. We adopted the ratio $R_1 = E(4_1^+)/E(2_1^+)$ for determining the affiliation of the nuclei. The nuclei $^{96}_{40}Zr$, $^{98}_{40}Z$, $^{98}_{42}Mo$ were distinguished by the fact that the ratio R_1 was smaller than the theoretically expected values for vibrational determination according to the collective model, which is $R_1 = 2$, as the ratio to the experimental values was equal to 1.57, 1.51, and 1.92 for the nuclei ^{96}Zr, ^{98}Zr, and ^{98}MO, respectively. The theoretical values of R_1, R_2, R_3 determined using IBM-2 were close to the corresponding experimental values as shown in Table (1). The rest nuclei, in which the energy level 0_2^+ appears as the first excited level:

$^{90}_{40}Zr, ^{96}_{32}Ge, ^{40}_{20}Ca, ^{16}_{8}O, and\ ^{4}_{2}He$, are characterized by the fact that the ratio $E(4_1^+)/E(2_1^+)$ is very small, it is equal to 1.25 for ^{90}Zr, 2 for ^{72}Ge ,1.35 for ^{40}Ca, 1.49 for ^{16}O. The ratio in germanium is considered an ideal value. We also noticed that these nuclei are characterized by the fact that the difference between the number of proton and neutron bosons equals one, and when the difference increases, the value of the ratio $E(4_1^+)/E(2_1^+)$ increases. The reason for the occurrence of this condition and the appearance of energy level 0_2^+ as the first excited level may be because these nuclei have double subshell closure. This kind of energy level is considered an abnormal state for the vibrational nuclei.

Table1. *Energy Ratio*

Isotopes	$R_1 = E(4_1^+)/E(2_1^+)$		$R_2 = E(6_1^+)/E(2_1^+)$		$R_3 = E(8_1^+)/E(2_1^+)$	
	Exp.	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**	**IBM-2**
^{96}Zr	1.571	1.529	1.974	1.839	2.507	2.020
^{98}Zr	1.507	1.556	2.036	2.552	2.630	3.143
^{98}Mo	1.917	2.162	2.976	3.491	4.155	4.987

6. Hamiltonian Parameters

The parameters of the energy Hamiltonian of the IBM-2 model are shown in Table (2). The distinctive observation in the Table (2) is the electric quadrupole interaction (κ) values were very small for all nuclei $\kappa \leq 0.01$. This parameter represents a measure of the deformation of the nucleus. So, its very small values indicate that the nuclei in this study don't suffer from deformation, so they have a regular spherical shape, which is symmetrical in all axes. Thus, we conclude that all energy levels are the result of the vibrational motion of the nuclei studied, and this gives further evidence for the appearance of the energy level (0_2^+) as the first excited level because this level (0_2^+) belongs to the beta vibrational band.

Table2. *IBM-2 Hamiltonian Parameters*

Isotopes	n_π	n_ν	ε	κ	χ_ν	χ_π	C_ν^0	C_ν^2	C_ν^4	C_π^0	C_π^2	C_π^4	ξ_1,ξ_3	ξ_2
^{96}Zr	5	3	1.7	-0.01	0.08	0.08	-0.3	0.0	0.099	-0.3	0.0	2.99	0.08	-2.99
^{98}Zr	5	4	0.99	-0.009	0.08	0.08	-0.5	-0.01	0.01	-0.5	-0.13	0.95	0.08	-0.95
^{98}Mo	4	3	0.7	-0.01	0.08	0.08	-0.7	-0.1	0.2	-0.7	0.2	0.25	0.3	0.3

7. ENERGY LEVELS

In our study, the theoretical values of energy levels were determined by applying the Hamiltonian of interacting bosons (IBM-2), after selecting the appropriate parameters for each isotope. Figures 1a,1b, and 1c) view a comparison between the experimental and theoretical values of the energy levels, where the energy levels have been categorized into three groups called: the ground band, beta, and gamma band. The results demonstrate acceptable agreement between the values and for all energy bands. The inconsistencies to experimental data in the energy level 0_2^+ are 0.2MeV, 0.009MeV and 0.18 MeV for ^{96}Zr, ^{98} Zr, and ^{98}MO respectively.

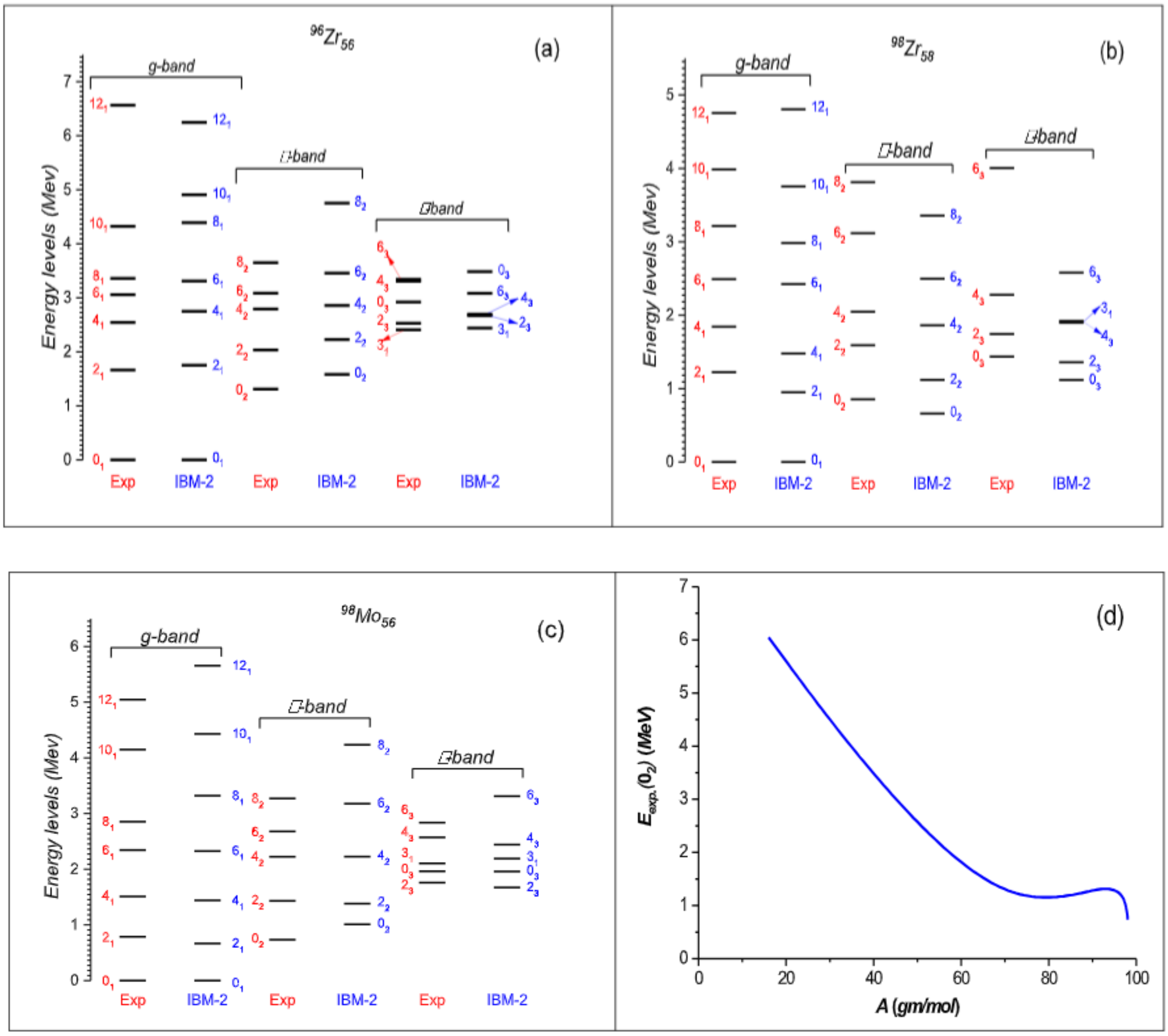

Fig1. *Energy levels of nuclei (a) Zirconium-96, (b) Zirconium-98, and (c) Molybdenum-98 estimated using the IBM-2 model and compared with experimental data. (d) Energy levels of 0_2 as a function of the atomic number in the nuclei* $^{98}_{42}\boldsymbol{Mo}, ^{98}_{40}\boldsymbol{Z}, ^{96}_{40}\boldsymbol{Zr}, ^{90}_{40}\boldsymbol{Zr}, ^{96}_{32}\boldsymbol{Ge}, ^{40}_{20}\boldsymbol{Ca},, ^{16}_{8}\boldsymbol{O}$

The correlation between the experimental energy levels of 0_2^+ and the mass numbers (A) of the seven mentioned nuclei is illustrated in Fig. 1-d. The results show that the energy of level 0_2^+ increases as the mass number decreases. In this context, level 0_2^+ serves as the first excited level. Seven isotopes in nature have a double-closed shell. Three of them are closed with a magic number, namely$^{90}_{40}Zr, ^{40}_{20}Ca,, ^{16}_{8}O$.The results show that level 0_2^+ has high energy, the reason possibly due to the double-closed shell with a magic number, where the binding energy is very high. For isotopes with a valence shell farther away from the magic number, the energy level of 0_2^+ has lower values. On the other hand, as the nucleus approaches a spherical (vibrational) shape, the energy level increases. Conversely, when it deviates from the spherical shape, due to deformation the energy level value decreases. The subsequent equation articulates the connection between the energy level $E(0_2^+)$ and the mass number: $E(0_2^+) = C \times ae^{-bA}$, where 'C' denotes a calibration constant, while 'a' and 'b' represent arbitrary constants.

8. ELECTROMAGNETIC TRANSITION PROBABILITY

(i) Electric quadrupole moment operators of neutron and proton boson (Q_π&Q_ν) are calculated using *NPBTRN* compiler. The matrix elements of electric transition $T^{(E2)}$ and electrical transition probability B(E2) are determined by equations (3&4). The experimental and estimated transitions via IBM-2 are listed in Table (3). The electric quadruple probabilities are overall very weak; the only significant transition between $2_1 \rightarrow 0_1$ (0.605 e^2b^2)in ^{96}Zr & ^{98}Zr, and between $2_2 \rightarrow 2_1$ (0.973 e^2b^2) in ^{98}Mo.

(ii) Magnetic transition probabilities$B(M1)$ were determined via equation(5), and the boson gyromagnetic factors were set to microscopic values $g_\pi = 0.285\ \mu_N$ & $g_\nu = 0.207\ \mu_N$ for Molybdenum nucleus, and $g_\pi = 0.147\ \mu_N$ & $g_\nu = 0.472\ \mu_N$ for Zirconium nucleus.

The outcomes of experimental and IBM-2 calculations of magnetic dipole transitions are listed in Table (4). In general, the values for the studied nuclei are very weak, which is attributed to their high stability.

Table3. *Electric Quadrupole Transitions in units of (e^2b^2)*

Transition	^{96}Zr		^{98}Zr		^{98}Mo	
$J_i^+ \rightarrow J_f^+$	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**
$2_1 \rightarrow 0_1$	0.60501	0.67804	0.93037	0. 99123	0.05652	0.05390
$2_1 \rightarrow 0_2$	0.43050	-	0.09864	0.09912	0.25450	0.26657
$2_2 \rightarrow 0_1$	0.00008	0.00005	0.64357	-	0.038158	0.04292
$2_2 \rightarrow 0_2$	0.47896	0.18776	0.00017	-	0.00594	0.00616
$2_2 \rightarrow 2_1$	0.00050	0.00042	0.00047	-	0.97333	0.15271
$2_3 \rightarrow 2_1$	0.04659	0.05215	0.00004	-	0.01409	0.01070
$2_3 \rightarrow 2_2$	0.00004		0.00001	-	0.19388	0.1590
$3_1 \rightarrow 2_1$	0.00835	0.00808	0.00003	-	0.00009	-
$3_1 \rightarrow 2_2$	0.18248	-	0.00001	-	0.15815	-
$3_1 \rightarrow 4_1$	0.00002	-	0.15307	-	0.65459	-
$4_1 \rightarrow 2_1$	0.34258	-	0.01522	0.08573	0.28917	0.13734
$4_2 \rightarrow 2_2$	0.04778	0.03129	0.2966	0.19290	0.18473	-
$6_1 \rightarrow 4_1$	0.01168	-	0.35192	0.38398	0.72257	-
$0_3 \rightarrow 2_1$	0.00008	-	0.09033	0.13390	0.00002	-
$0_3 \rightarrow 2_2$	0.028070	0.06510	0.07255	-	0.00003	-

Table4. *Magnetic Dipole Transitions in units of (μ_N^2)*

Transition $J_i^+ \rightarrow J_f^+$	^{96}Zr		^{98}Zr		^{98}Mo	
	IBM-2	**Exp.**	**IBM-2**	**Exp.**	**IBM-2**	**Exp.**
$2_1^+ \rightarrow 0_1^+$	0.00000	0.00000	0.00000	0.00000	0.00000	-
$2_2^+ \rightarrow 0_1^+$	0.00000	0.00000	0.00000	0.00000	0.00000	-
$2_2^+ \rightarrow 2_1^+$	0.00211	0.00616	0.46806	-	0. 77915	-
$2_3^+ \rightarrow 2_1^+$	0.00002	0.00002	0.00005	-	0.00200	0.0032
$2_3^+ \rightarrow 2_2^+$	0.08973	0.08400	0.00001	-	0.00070	0.00020
$2_4^+ \rightarrow 2_1^+$	0.04975	-	0. 00009	-	0.00216	-
$2_4^+ \rightarrow 2_2^+$	0.00003	-	0.00006	-	0.02164	-

(iii) Electric Monopole Transitions (E0) occur when there is no change in angular momentum between initial and final nuclear states and no parity change. Specifically, single gamma emission is strictly forbidden for spin-zero to spin-zero transitions. The term "electric monopole" refers to the absence of angular momentum change, making it a unique type of transition.

Electric monopole transitions (E0) were determined as follows [25]:

$$B(E0; J_i \rightarrow J_f) = e^2R^4\rho^2(E0), \quad where\ J_i = J_f$$
$$= 2.74 \times 10^{-4}A^{4/3}\rho^2(E0) \ \cdots\cdots\cdots (7)$$

where R is the radius defined as $R=R_0A^{1/3}$and$\rho(E0)$ dimensionless monopole transition strength parameter defined as follows $\rho(E0) = \frac{\langle 0_f^+|M(E0)|0_i^+\rangle}{eR^2}$

Table5. *Electric Monopole Transitions in units of (e^2b^2)*

Isotopes	$J_i^+ \rightarrow J_f^+$	**IBM-2**	**Exp.**
^{96}Zr	$0_2 \rightarrow 0_1$	0.0516	-

^{98}Zr	$0_2 \rightarrow 0_1$	0.00001	0.000006
^{98}Mo	$0_2 \rightarrow 0_1$	0.1460	-

9. NUCLEI'S SURFACE SHAPE

By computing the surface energy value, researchers gain valuable insights into the stability and deformation of atomic nuclei, providing crucial information for understanding nuclear structure and behavior within the interacting boson model framework [26].

$$E = \varepsilon N \frac{\beta^2}{1+\beta^2} - |\kappa| \left[\frac{N[5+(1+\chi^2)\beta^2]}{1+\beta^2} + \frac{N(N-1)}{(1+\beta^2)^2} \left(\frac{2}{7}\chi^2\beta^4 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos(3\gamma) + 4\beta^2 \right) \right]$$

To calculate surface energy using the equation above, one must first determine the parameters' values. These parameters include the number of bosons (N) and deformation parameters (β, γ, χ), which characterize the structural properties of the nucleus. Figure (2) shows the relationship between the surface energy and the deformation parameters, figures show there is no deformation in the nuclei ^{96}Zr, ^{98}Zr, and ^{98}Mo respectively.

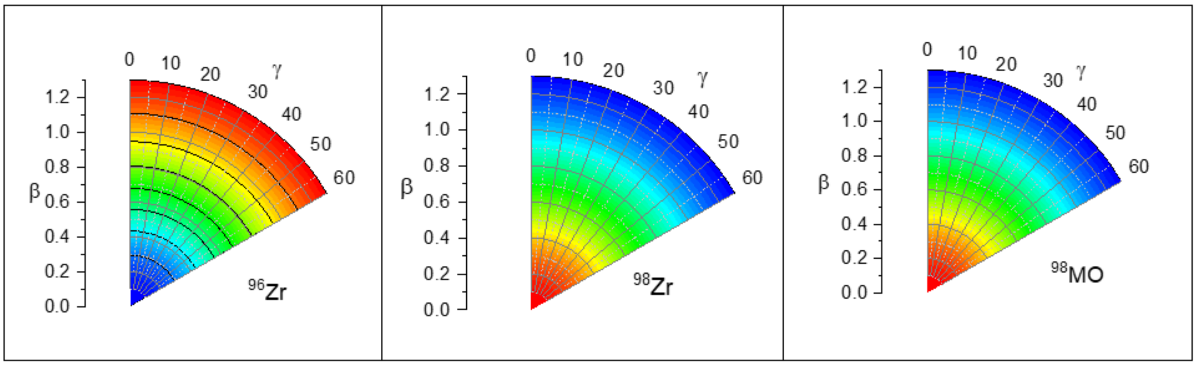

Fig2. *Surface energy as a function of deformation parameters ($\boldsymbol{\beta}, \boldsymbol{\gamma}$) gives a prediction about the shape of nuclei ^{96}Zr, ^{98}Zr, and ^{98}Mo.*

10. CONCLUSIONS

The nuclei exhibited very small R-ratio values, classifying them as vibrational nuclei. The observed weak electromagnetic transition values suggest a high degree of nuclear stability. The regular surface shape of these nuclei clearly indicates their spherical nature. The high stability of these isotopes can be attributed to their double-closed shell structure. The IBM-2 model proved to be highly effective in accurately describing and studying these nuclei. The presence of these intruder levels, positioned differently from their expected locations as predicted by the IBM model, can be explained by their double subshell closure. Additionally, the estimated values for electric quadrupole transitions, magnetic dipole transitions, and zero transitions were in acceptable agreement with the experimental data. Overall, this study enhances our understanding of nuclear structure and stability, particularly in the context of intruder nuclear levels. The successful application of the IBM-2 model underscores its utility in investigating such rare and complex nuclear phenomena.

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