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Evaluation of the Cosmological Constant from de Broglie Pilot-Wave Dynamics: Inflation with Minimal Coupling

Jung-Jeng Huang

Department of Mechanical Engineering, Physics Division
Ming Chi University of Technology
Taishan, New Taipei City, Taiwan
huangjj@mail.mcut.edu.tw

Abstract: *We evaluate the cosmological constant from the point of view of the de Broglie-Bohm pilot-wave theory. In Schrödinger picture we study the effects of trans-Planckian physics on the de Broglie-Bohm quantum trajectory of massless minimally coupled scalar field in de Sitter space. For the Corley-Jacobson type dispersion relations with quartic or sextic correction, it is shown that through de Broglie's first-order dynamics, there exists a transition in the evolution of the quantum trajectory from well before horizon exit to horizon exit, providing a possible mechanism for generating a small cosmological constant. Moreover, comparing the trans-Planckian effects of both quartic and sextic corrections on the quantum trajectory, we find that the latter is much smaller than the former. We also show how the cosmological constant reduces during the slow-roll inflation at the grand unification phase transition through calculating explicitly the finite vacuum energy density due to fluctuations of the inflaton field. Finally, we suggest the possibility that a small current value of the cosmological constant could be obtained through cascade transition in the evolution of the Universe.*

Keywords: *De Broglie-Bohm trajectory, inflation, cosmological constant, trans-Planckian physics.*

1. INTRODUCTION

The cosmological constant problem [1] is one of the outstanding theoretical challenges in modern physics. While cosmic acceleration [2, 3] suggests that the Universe has a small positive cosmological constant ($\Lambda \cong 1.18\times10^{-123}{M_{Pl}}^4$ in Planck units, $M_{Pl} = G^{-1/2} = 1.22\times10^{19}$ GeV is the Planck mass), we expect a cosmological constant of order ${M_{Pl}}^4$ from an effective quantum field theory up to the Planck scale. Therefore the discrepancy between the observational and theoretical values is of 123 orders of magnitude.

Actually before the Big Bang the Universe was also in an era of inflation with an exponentially increasing scale factor. In the standard inflationary scenario, usual realization of inflation is associated with a slow rolling inflaton minimally coupled to gravity [4]. However, standard inflationary predictions can have two extensions. The first extension is associated with the ambiguity of initial quantum vacuum state, and the choice of initial vacuum state affects the predictions of inflation [5, 6]. For example, a deterministic hidden-variables theory such as the de Broglie-Bohm pilot-wave theory [7, 8] allows the existence of vacuum states with non-standard or non equilibrium field fluctuations [9, 10], which result in statistical predictions that deviate from those of quantum theory in the context of inflationary cosmology [11, 12]. Recent study also shows that the quantum-to-classical transition of primordial cosmological perturbations can be obtained in the context of the de Broglie-Bohm theory [13].

The second extension concerns the so-called trans-Planckian problem [14, 15] of whether the predictions of standard cosmology are insensitive to the effects of trans-Planckian physics. In fact, nonlinear dispersion relations such as the Corley-Jacobson (CJ) type were used to mimic the trans-Planckian effects on cosmological perturbations [14-16]. These CJ type dispersion relations can be obtained naturally from quantum gravity models such as Horava gravity [17, 18]. Moreover, in several approaches to quantum gravity, the phenomenon of running spectral dimension of spacetime from the standard value of 4 in the infrared to a smaller value in the ultraviolet is associated with modified dispersion relations, which also include the CJ type dispersion relations [19, 20].

In the previous work [21-25] we used the lattice Schrödinger picture to study the free scalar field theory in de Sitter space, derived the wave functionals for the Bunch-Davies (BD) vacuum state and its excited states, and found the trans-Planckian effects on the de Broglie-Bohm quantum trajectory of massless minimally coupled scalar field for the CJ type dispersion relations with sextic correction through Bohm's second-order dynamics. In this paper we try to study the trans-Planckian effects on the quantum trajectories of scalar field for the CJ type dispersion relations with quartic or sextic correction through de Broglie's first-order dynamics.

The paper is organized as follows. In section 2, the de Broglie-Bohm pilot-wave theory of massless minimally coupled scalar field in de Sitter space is briefly reviewed in the Schrödinger picture, and the de Broglie-Bohm quantum trajectories for scalar field with linear dispersion relation are given. In section 3, we study the effects of trans-Planckian physics on the massless minimally coupled scalar field during the slow-roll inflation, and use the CJ type dispersion relations with quartic or sextic correction to obtain the time evolution of the vacuum state wave functional and the corresponding quantum trajectories through de Broglie's first-order dynamics. In section 4, using the results of section 3, we calculate the finite cacuum energy density due to fluctuations of the inflaton field and use the back reaction to address the cosmological constant problem. Finally, conclusion and discussion are presented in section 5. In this paper we will set $\hbar$=c=1.

2. DE-BROGLIE-BOHM PILOT-WAVE THEORY OF SCALAR FIELD

In this section, we begin by briefly reviewing how to define the de Broglie-Bohm pilot-wave theory of massless minimally coupled scalar field in de Sitter space in the Schrödinger picture (for the details see [25]). We consider the Lagrangian density for the scalar field

$$L = |g|^{\frac{1}{2}} \left\{ \frac{1}{2} \left[g^{\mu\nu}(x)\varphi(x)_{,\mu}\varphi(x)_{,\nu} \right] \right\}, \tag{1}$$

where ϕ is a real scalar field, g = det $g_{\mu\nu}$, μ,ν=0, 1, 2, 3. For a spatially flat (1+3)-dimensional Robertson-Walker spacetime with scale factor $a(t)$, we have

$ds^2 = dt^2 - a^2(t)d^2x^i,\ i = 1,2,3,$

$$L = a^3 \left\{ \frac{1}{2} \left[(\partial_0\varphi)^2 - a^{-2}(\partial_i\varphi)^2 \right] \right\}. \tag{2}$$

In the (1+3)-dimensional de Sitter space we have $a(t) = \exp(ht)$, where $h \equiv \dot{a}/a$ is the Hubble parameter which is a constant.

In the Schrödinger picture, from (2) we can obtain the time-dependent functional Schrödinger equation in momentum space [25]

$$i\frac{\partial\psi}{\partial t} = \sum_{rk}\{ -\frac{1}{2}\frac{\partial^2}{\partial\varphi_{rk}{}^2} + \frac{1}{2}\left[a^{-2}k^2 - \frac{9}{4}h^2 \right]\varphi_{rk}{}^2\}\psi\ , \tag{3}$$

$$\psi\left[\varphi_{rk}, t\right] \equiv \prod_{rk}\psi_{rk}(\varphi_{rk}, t)\ . \tag{4}$$

Here, r =1, 2, $k \equiv (k_1, k_2, k_3)$ denotes momentum, ϕ_{1k} and ϕ_{2k} are the real and imaginary parts of ϕ_k respectively, *i.e.* $\phi_k = \phi_{1k} + i\phi_{2k}$. For each real mode ϕ_{rk}, we have

$$i\frac{\partial\psi_{rk}}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi_{rk}}{\partial\phi_{rk}{}^2} + \frac{1}{2}\left[a^{-2}k^2 - \frac{9}{4}h^2 \right]\phi_{rk}{}^2\psi_{rk}\ . \tag{5}$$

Thus (5) governs the time evolution of the state wave functional $|\psi_{rk}\rangle$ in the { $|\phi_{rk}\rangle$ } representation. In terms of the conformal time τ defined by

$$d\tau = dt/a\,,\, \tau = -h^{-1}\exp\left(-ht\right) = -h^{-1}a^{-1},\ -\infty < \tau < 0, \tag{6}$$

the normalized vacuum and its excited states are

$$\psi_{rk(n_{rk})}(\varphi_{rk},\tau) = R_{(n_{rk})}(\varphi_{rk},\tau)\exp\left(i\Theta_{n_{rk}}\left(\varphi_{rk},\tau\right)\right), n_{rk} = 0,1,2,... \tag{7}$$

with the amplitude $R_{(n_{rk})}(\phi_{rk},\tau)$ and phase $\Theta_{(n_{rk})}(\phi_{rk},\tau)$

$$R_{(n_{rk})}(\phi_{rk},\tau) = \left[\frac{\sqrt{2h/\pi}}{\sqrt{\pi}\,2^{n}n_{rk}!\left|H_{3/2}{}^{(1)}\right|}\right]^{1/2} H_{(n_{rk})}(\eta_{rk})\exp(-\frac{1}{2}\eta_{rk}{}^{2}), \tag{8}$$

$$\Theta_{(n_{rk})}(\varphi_{rk},\tau) = -\frac{hk|\tau|}{2}\frac{\left|H_{3/2}{}^{(1)}\right|'}{\left|H_{3/2}{}^{(1)}\right|}\varphi_{rk}{}^{2} - (\frac{1}{2}+n_{rk})\int\frac{\frac{2}{\pi|\tau|}}{\left|H_{3/2}{}^{(1)}\right|^{2}}d\tau\,. \tag{9}$$

Here, η_{rk} is defined by $\eta_{rk} \equiv \left(\sqrt{2h/\pi}\,/\left|H_{3/2}{}^{(1)}\right|\right)\varphi_{rk}$, $H_{(n_{rk})}(\eta_{rk})$ is the nth-order Hermite polynomial, $H_{3/2}{}^{(1)}(k|\tau|)$ is the Hankel function of the first kind of order $3/2$, and the prime in (9) denotes the derivative with respect to $k|\tau|$. The complete wave functionals are $\psi_{[n]}\left[\varphi_{rk},t\right] = \prod_{rk}\psi_{(n_{rk})}(\varphi_{rk},t)$, where $[n]\equiv(n_i,n_j,\cdots)$ means that mode i is in the n_i excited state, mode j is in the n_j excited state, *etc.* For $n_{rk}=0$, the ground state wave functional corresponds to the BD vacuum.

Note that (3) implies the continuity equation

$$\frac{\partial|\psi|^{2}}{\partial t} + \sum_{rk}\{\frac{\partial}{\partial\phi_{rk}}\left[|\psi|^{2}\frac{\partial\Theta}{\partial\phi_{rk}}\right]\} = 0 \tag{10}$$

and the de Broglie-Bohm velocity field

$$\frac{d\phi_{rk}}{dt} = \frac{\partial\Theta}{\partial\phi_{rk}}, \tag{11}$$

where $\psi = |\psi|\exp\left(i\Theta\right)$. For a single mode ϕ_{rk}, we have $\psi_{rk} = |\psi_{rk}|\exp\left[i\Theta_{rk}\right]$ with $\Theta = \sum_{rk}\Theta_{rk}$,

the continuity equation

$$\frac{\partial|\psi_{rk}|^{2}}{\partial t} + \frac{\partial}{\partial\phi_{rk}}\left(|\psi_{rk}|^{2}\frac{\partial\Theta_{rk}}{\partial\phi_{rk}}\right) = 0\,, \tag{12}$$

and the de Broglie-Bohm velocity field

$$\frac{d\phi_{rk}}{dt} = \frac{\partial\Theta_{rk}}{\partial\phi_{rk}}\,. \tag{13}$$

Here, ψ is interpreted as a physical field in field configuration space, guiding the evolution of ϕ_{rk} through (3) and (13). Substituting (9) into (13) and using τ gives

$$\frac{d\varphi_{rk}}{d\tau} = -k\frac{\left|H_{3/2}{}^{(1)}(k|\tau|)\right|'}{\left|H_{3/2}{}^{(1)}(k|\tau|)\right|}\varphi_{rk}\,, \tag{14}$$

which yields the quantum trajectory

$$\phi_{rk}(z) = C\left|H_{3/2}{}^{(1)}(z)\right|, \tag{15}$$

where $z \equiv k|\tau| = \ k/a \ /h$ is the ratio of physical wave number $k_{phys} \equiv k/a$ to the inverse of Hubble radius, and C is an integration constant.

3. TRANS-PLANCKIAN EFFECTS

In this section we study the effects of trans-Planckian physics on the massless minimally coupled scalar field in the slow-roll inflation. Particularly, we use the CJ type dispersion relations

$$\omega^2(k/a) = k^2\left[1 + b_s(\frac{k}{aM})^{2s}\right], \tag{16}$$

where M is a cutoff scale, s is an integer, and b_s is an arbitrary coefficient [13-15].

3.1. CJ Type Dispersion Relations with Quartic Correction

We first focus on the CJ type dispersion relations (16) with $s = 1$ and $b_1 > 0$. Notice that these CJ type dispersion relations can be obtained from theories based on quantum gravity models [17-20].

3.1.1. Evolution of Vacuum Wave Functional

Using $z = k|\tau| = k/ah$, (3) then becomes

$$i\frac{\partial\psi}{\partial t} = \sum_{rk}\{-\frac{1}{2}\frac{\partial^2}{\partial\varphi_{rk}{}^2} + \frac{1}{2}\left[z^2 \ \ 1+\sigma^2 z^2 \ \ h^2 - \frac{9}{4}h^2\right]\varphi_{rk}{}^2\}\psi, \tag{17}$$

where $\sigma^2 \equiv b_1(h/M)^2$, and the corresponding ground state wave functional of (17) is

$$\psi_{(0)} = \prod_{rk} A_{k(0)}(\tau)\exp(-\frac{1}{2}B_k(\tau)a^{-1}\phi_{rk}{}^2), \tag{18}$$

where $A_{k(0)}(\tau)$ and $B_k(\tau)$ satisfy respectively

$$A_{k(0)}(\tau) = \exp\left[-i\frac{1}{2}\int B_k(\tau)d\tau + const\right], \tag{19}$$

$$B_k{}^2(\tau) - i\left[\frac{dB_k(\tau)}{d\tau} + \frac{B_k(\tau)}{\tau}\right] - \left[k^2(1+\sigma^2 z^2) - \frac{9}{4\tau^2}\right] = 0. \tag{20}$$

In region I where $k_{phys} \equiv k/a > M$, i.e., $z > M/h$, the dispersion relation can be approximated by $\omega^2(k/a) \approx k^2\sigma^2 z^2$, and the corresponding wave functional for the initial BD vacuum state is [25, 26]

$$\psi_{(0)}{}^{\mathrm{I}} = \prod_{rk} A_{k(0)}{}^{\mathrm{I}}(\tau)\exp(-\frac{1}{2}B_k{}^{\mathrm{I}}(\tau)a^{-1}\phi_{rk}{}^{\mathrm{I}2}),$$

$$A_{k(0)}{}^{\mathrm{I}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k{}^{\mathrm{I}}(\tau)d\tau + const\right], \tag{21}$$

$$B_k{}^{\mathrm{I}}(\tau) = \frac{\dfrac{4}{\pi|\tau|}}{\left|H_{3/4}{}^{(1)}\right|^2} - i\frac{k}{2}\frac{\left(\left|H_{3/4}{}^{(1)}\right|^2\right)'}{\left|H_{3/4}{}^{(1)}\right|^2}\sigma z, \tag{22}$$

where the prime in (22) denotes the derivative with respect to $\sigma z^2/2$.

On the other hand, in region II where $k_{phys} \equiv k/a < M$, i.e., $z < M/h$, linear relation recovers $\omega^2 \cong k^2$, and the corresponding wave functional for the non-BD vacuum state is [25, 26]

$$\psi_{(0)}{}^{\mathrm{II}} = \prod_{rk} A_{k(0)}{}^{\mathrm{II}}(\tau)\exp(-\frac{1}{2}B_k{}^{\mathrm{II}}(\tau)a^{-1}\phi_{rk}{}^{\mathrm{II}2}),$$

$$A_{k(0)}{}^{\mathrm{II}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k{}^{\mathrm{II}}(\tau)d\tau + const\right], \tag{23}$$

$$B_k{}^{\mathrm{II}}(\tau) = \frac{\dfrac{2}{\pi|\tau|}}{\left(\left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2\right)\left|H_{3/2}{}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1{}^{\mathrm{II}}C_2{}^{\mathrm{II}*}\ H_{3/2}{}^{(1)\ 2}\right]}$$

$$-i\frac{k}{2}\frac{\left\{\left(\left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2\right)\left|H_{3/2}{}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1{}^{\mathrm{II}}C_2{}^{\mathrm{II}*}\ H_{3/2}{}^{(1)\ 2}\right]\right\}'}{\left(\left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2\right)\left|H_{3/2}{}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1{}^{\mathrm{II}}C_2{}^{\mathrm{II}*}\ H_{3/2}{}^{(1)\ 2}\right]}, \tag{24}$$

where the prime in (24) denotes the derivative with respect to z, and the constants $C_1{}^{\mathrm{II}}$ and $C_2{}^{\mathrm{II}}$ satisfy $\left|C_1{}^{\mathrm{II}}\right|^2 - \left|C_2{}^{\mathrm{II}}\right|^2 = 1$. Let τ_c be the time when the modified dispersion relations take the standard linear form. Then we have $\sigma^2 z_c{}^2 = 1$ where $z_c = k|\tau_c| = M/b_1^{1/2}h >> 1$ for $b_1 \sim 1$. Note that $C_1{}^{\mathrm{II}}$ and $C_2{}^{\mathrm{II}}$ can be obtained by the following matching conditions at τ_c for the two wave functionals (21) and (23)

$$\psi_{(0)}{}^{\mathrm{I}}\Big|_{z_c} = \psi_{(0)}{}^{\mathrm{II}}\Big|_{z_c}, \tag{25}$$

$$\frac{d\psi_{(0)}{}^{\mathrm{I}}}{dz}\Big|_{z_c} = \frac{d\psi_{(0)}{}^{\mathrm{II}}}{dz}\Big|_{z_c}, \tag{26}$$

which can also be rewritten respectively as

$$\mathrm{Re}\ B_k{}^{\mathrm{I}}\ \Big|_{z_c} = \mathrm{Re}\ B_k{}^{\mathrm{II}}\ \Big|_{z_c}, \tag{27}$$

$$\frac{d\,\mathrm{Re}\ B_k{}^{\mathrm{I}}}{dz}\Big|_{z_c} = \frac{d\,\mathrm{Re}\ B_k{}^{\mathrm{II}}}{dz}\Big|_{z_c}, \tag{28}$$

by requiring $B_k{}^{\mathrm{I}} = B_k{}^{\mathrm{II}}$, $\phi_{rk}{}^{\mathrm{I}} = \phi_{rk}{}^{\mathrm{II}}$ and $A_{k(0)}{}^{\mathrm{I}} = A_{k(0)}{}^{\mathrm{II}}$ when $z = z_c$.

Using $\left|H_{3/4}{}^{(1)}\ \sigma z^2/2\ \right|^2 = 4/\pi\sigma z^2\ \ 1 + 5/8\sigma^2 z^4 + \ldots\ \approx 4/\pi\sigma z^2$ with $\sigma = z_c{}^{-1}$, $z_c >> 1$ and $\left|H_{3/2}{}^{(1)}(z)\right|^2 = z^{-3}(1+z^2)$, we obtain from (22), (24), and (27)

$$1 = \left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2 + 2\left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\cos(2z_c - \theta). \tag{29}$$

Here we choose $C_1{}^{\mathrm{II}} = \left|C_1{}^{\mathrm{II}}\right|$ and $C_2{}^{\mathrm{II}} = \left|C_2{}^{\mathrm{II}}\right|\exp(i\theta)$, and θ is a relative phase parameter. Then from (29) and $\left|C_1{}^{\mathrm{II}}\right|^2 - \left|C_2{}^{\mathrm{II}}\right|^2 = 1$ we have

$$\left|C_1^{\mathrm{II}}\right| = \csc\left(2z_c - \theta\right), \ \left|C_2^{\mathrm{II}}\right| = -\cot(2z_c - \theta), \tag{30}$$

where $\sin(2z_c - \theta) > 0$, $\cos(2z_c - \theta) < 0$. Substituting (22) and (24) into (28) and keeping terms up to order $1/z_c$ on the right-hand side of (28), we find

$$\frac{1}{z_c} = \left|C_1^{\mathrm{II}}\right|\left|C_2^{\mathrm{II}}\right|\cos(2z_c - \theta)\frac{8}{z_c} + 4\left|C_1^{\mathrm{II}}\right|\left|C_2^{\mathrm{II}}\right|\sin\left(2z_c - \theta\right). \tag{31}$$

Using (30) in (31) yields

$$\cot(2z_c - \theta) = -\frac{1}{4z_c} \ \text{ or } \ \cot(2z_c - \theta) = -\frac{z_c}{2} + \frac{1}{4z_c}. \tag{32}$$

Here we choose $\cot(2z_c - \theta) = -1/4z_c$, so that $\left|C_2^{\mathrm{II}}\right|$ is small for z_c >>1 to avoid an unacceptably large back reaction on the background geometry. Therefore we have

$$\left|C_2^{\mathrm{II}}\right| = \frac{1}{4z_c}, \left|C_1^{\mathrm{II}}\right| = \sqrt{1 + \left|C_2^{\mathrm{II}}\right|^2} \cong 1 + \frac{1}{32 z_c^{\ 2}} \cong 1, \tag{33}$$

or $\quad \sin(2z_c - \theta) \cong 1, \ \cos(2z_c - \theta) \cong -\dfrac{1}{4z_c}.$ (34)

3.1.2. De Broglie-Bohm Quantum Trajectory

In section 2, we defined the pilot-wave scalar field theory through de Broglie's first-order dynamics (3) and (13). Using the result about the evolution of vacuum state in subsection 3.1.1, we can further define it through Bohm's second-order dynamics (17) and (35)

$$\frac{d^2\phi_{rk}}{dt^2} = -\frac{\partial}{\partial\phi_{rk}}(V + Q). \tag{35}$$

Here the classical potential V and the so-called 'quantum potential' Q are given by

$$V = \sum_{rk}\frac{1}{2}\left[z^2\left(1 + \sigma^2 z^2\right)h^2 - \frac{9}{4}h^2\right]\varphi_{rk}^{\ 2}, \tag{36}$$

$$Q = -\sum_{rk}\frac{1}{2\left|\psi_{(0)}\right|}\frac{\partial^2\left|\psi_{(0)}\right|}{\partial\phi_{rk}^{\ 2}}, \tag{37}$$

where $\psi_{(0)}$ is given by (18)-(20) and $\left|\psi_{(0)}\right|$ is given by (8) with $n_{rl} = 0$. Note that Bohm's dynamics in general yields more possible quantum trajectories than de Broglie's dynamics does [24], and this distinction between Bohm's and de Broglie's dynamics was also emphasized by Valentini. This is what we expect, because Bohm regarded (35) as the law of motion, with the de Broglie guidance equation (13) added as a constraint on the initial momenta.

However, recently it was pointed out that Bohm's second-order dynamics is unstable. Small deviations from initial quantum equilibrium do not relax and instead grow with time [27]. On the other hand, de Broglie's first-order dynamics is a tenable physical theory. Therefore, we will investigate the quantum trajectories of scalar field through de Broglie's dynamics hereafter.

In region I, from (21) and (22) we have

$$\Theta_{(0)}(\phi_{rk}^{\ \mathrm{I}}, \tau) = -\frac{k}{4}\frac{\left(\left|H_{3/4}^{\ (1)}\right|^2\right)'}{\left|H_{3/4}^{\ (1)}\right|^2}\sigma z a^{-1}\phi_{rk}^{\ \mathrm{I}2} - \frac{1}{2}\int\frac{\frac{4}{\pi|\tau|}}{\left|H_{3/4}^{\ (1)}\right|^2}d\tau, \tag{38}$$

where $\left|H_{3/4}^{(1)}\left(\sigma z^2/2\right)\right|^2 = \left(4/\pi\sigma z^2\right)\left(1+5/8\sigma^2 z^4+\ldots\right) \approx 4/\pi\sigma z^2$, and the prime in (38) denotes the derivative with respect to $\sigma z^2/2$. Substituting (38) into (13) and using $d\tau = dt/a$ and $z = k|\tau| = k/ah$ gives

$$\frac{d\phi_{rk}{}^{\mathrm{I}}}{d\tau} = \frac{k}{z}\phi_{rk}{}^{\mathrm{I}}. \tag{39}$$

The general solution of (39) is

$$\phi_{rk}{}^{\mathrm{I}} = \bar{C}^{\mathrm{I}} z^{-1}. \tag{40}$$

On the other hand, in region II, from (23) and (24) we have

$$\Theta_{(0)}(\phi_{rk}{}^{\mathrm{II}},\tau) = -\frac{k}{4}\frac{\left(\left|H_{3/2}^{(1)}\right|_{md}^{2}\right)'}{\left(\left|H_{3/2}^{(1)}\right|_{md}^{2}\right)} a^{-1}\varphi_{rk}{}^{\mathrm{II}2} - \frac{1}{2}\int \frac{\dfrac{2}{\pi|\tau|}}{\left(\left|H_{3/2}^{(1)}\right|_{md}^{2}\right)} d\tau, \tag{41}$$

where $\left|H_{3/2}^{(1)}\right|_{md}$ means $\left|H_{3/2}^{(1)}\right|$ modified according to

$$\left|H_{3/2}^{(1)}\right|_{md} \equiv \left\{\left(|C_1|^2+|C_2|^2\right)\left|H_{3/2}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1C_2^*\left(H_{3/2}^{(1)}\right)^2\right]\right\}^{1/2}, \tag{42}$$

$$\left|H_{3/2}^{(1)}(z)\right|^2 = \frac{2}{\pi z}\left(1+\frac{1}{z^2}\right), \tag{43}$$

and the prime in (41) denotes the derivative with respect to z. Note that in region II, $\left|H_{3/2}^{(1)}\right|_{md}$ Becomes

$$\left|H_{3/2}^{(1)}\right|_{md} = \left|H_{3/2}^{(1)}\right|\left\{\left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2 + 2\left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\left[\cos(2z-\theta)\frac{z^2-1}{z^2+1} - \sin(2z-\theta)\frac{2z}{1+z^2}\right]\right\}, \tag{44}$$

Which can be approximated by $\left|H_{3/2}^{(1)}\right|$ as z decreases from $z \to z_c >> 1$ (well before horizon exit) to $z=1$ (horizon exit) by using (29), (33), and (34). Therefore, substituting (41) into (13) and using $d\tau = dt/a$ and $z = k|\tau| = k/ah$ yields the general solution

$$\varphi_{rk}{}^{\mathrm{II}} = \bar{C}^{\mathrm{II}}\left|H_{3/2}^{(1)}(z)\right| = \bar{C}^{\mathrm{II}}\sqrt{2/\pi}\,z^{-3/2}\left(1+z^2\right)^{1/2}. \tag{45}$$

Then, substituting (40) and (45) into the matching condition at z_c for $\phi_{rk}{}^{\mathrm{I}}$ and $\phi_{rk}{}^{\mathrm{II}}$

$$\phi_{rk}{}^{\mathrm{I}}\Big|_{z_c} = \phi_{rk}{}^{\mathrm{II}}\Big|_{z_c} \tag{46}$$

and using $z_c >> 1$, we obtain

$$\bar{C}^{\mathrm{II}} = \sqrt{\pi/2}\,\bar{C}^{\mathrm{I}} z_c^{-1/2}. \tag{47}$$

Since in the 3-dimensional space ϕ_{rk} contains a factor $a^{3/2}$ which is proportional to $z^{-3/2}$, we can use a field redefinition $u_{rk} \equiv a^{-3/2}\phi_{rk}$, $a = (k/h)z^{-1}$, and (47) to rewrite (40) and (45) as

$$u_{rk}{}^{\mathrm{I}} = \left(\frac{k}{h}\right)^{-3/2} \overline{C}^{\mathrm{I}} z^{1/2},\; u_{rk}{}^{\mathrm{II}} = \left(\frac{k}{h}\right)^{-3/2} \overline{C}^{\mathrm{I}} z_c{}^{-1/2}(1+z^2)^{1/2}. \tag{48}$$

From (48) we see that for fixed k and $z_c >> 1$, as z decreases from $z = z_c >> 1$ to $z = 1$, the scalar field value reduces $z_c/\sqrt{2}$ times, i.e., there exists a transition in the time evolution of the quantum trajectory of scalar field.

3.2. CJ Type Dispersion Relations with Sextic Correction

In this subsection, we consider the CJ type dispersion relations (16) with $s = 2$ and $b_2 > 0$, and repeat the preceding calculations for these type dispersion relations.

3.2.1. Evolution of Vacuum Wave Functional

For this case, only (17), (20), and (22) are changed into

$$i\frac{\partial \psi}{\partial t} = \sum_{rk}\{-\frac{1}{2}\frac{\partial^2}{\partial \varphi_{rk}{}^2} + \frac{1}{2}\left[z^2\ \ 1+\sigma^2 z^4\ \ h^2 - \frac{9}{4}h^2\right]\varphi_{rk}{}^2\}\psi\,, \tag{49}$$

$$B_k{}^2(\tau) - i\left[\frac{dB_k(\tau)}{d\tau} + \frac{B_k(\tau)}{\tau}\right] - \left[k^2(1+\sigma^2 z^4) - \frac{9}{4\tau^2}\right] = 0\,, \tag{50}$$

$$B_k{}^{\mathrm{I}}(\tau) = \frac{\dfrac{6}{\pi|\tau|}}{\left|H_{1/2}{}^{(1)}\right|^2} - i\frac{k}{2}\frac{\left(\left|H_{1/2}{}^{(1)}\right|^2\right)'}{\left|H_{1/2}{}^{(1)}\right|^2}\sigma z^2\,, \tag{51}$$

where $\sigma^2 \equiv b_2(h/M)^4$, and the prime in (51) denotes the derivative with respect to $\sigma z^3/3$. Using $\left|H_{1/2}{}^{(1)}\ \ \sigma z^3/3\ \right|^2 = 6/\pi\sigma z^3$, $\sigma = \bar{z}_c{}^{-2}$, $\bar{z}_c = k|\bar{\tau}_c| = M/b_2{}^{1/4}h$ >>1 for $b_2 \sim 1$, and $\left|H_{3/2}{}^{(1)}(z)\right|^2 =\ \ 2/\pi\ \ z^{-3}(1+z^2)$, we obtain from (51), (24), (27), and $\left|C_1{}^{\mathrm{II}}\right|^2 - \left|C_2{}^{\mathrm{II}}\right|^2\ = 1$

$$1 = \left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2 + 2\left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\cos(2\bar{z}_c - \theta)\,, \tag{52}$$

$$\left|C_1{}^{\mathrm{II}}\right| = \csc\ \ 2\bar{z}_c - \theta\ \ ,\ \left|C_2{}^{\mathrm{II}}\right| = -\cot(2\bar{z}_c - \theta)\,, \tag{53}$$

where $\sin(2\bar{z}_c - \theta) > 0$, $\cos(2\bar{z}_c - \theta) < 0$. Substituting (51) and (24) into (28) and keeping terms up to order $1/\bar{z}_c$ on the right-hand side of (28), we find

$$\frac{2}{\bar{z}_c} = \left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\cos(2\bar{z}_c - \theta)\frac{8}{\bar{z}_c} +\ 4\left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\sin\ \ 2\bar{z}_c - \theta\ \ . \tag{54}$$

Using (53) in (54) yields

$$\cot(2\bar{z}_c - \theta) = -\frac{1}{2\bar{z}_c} \quad \text{Or}\ \cot(2\bar{z}_c - \theta) = -\frac{\bar{z}_c}{2} + \frac{1}{2\bar{z}_c}. \tag{55}$$

Here we choose $\cot(2\bar{z}_c - \theta) = -\dfrac{1}{2\bar{z}_c}$, so that $\left|C_2{}^{\mathrm{II}}\right|$ is small for $\bar{z}_c$ >>1 to avoid an unacceptably large back reaction on the background geometry. Therefore we have

$$\left|C_2{}^{\mathrm{II}}\right| = \frac{1}{2\bar{z}_c}\,,\ \left|C_1{}^{\mathrm{II}}\right| = \sqrt{1+\left|C_2{}^{\mathrm{II}}\right|^2} \cong 1 + \frac{1}{8\bar{z}_c{}^2} \cong 1\,, \tag{56}$$

or $$\sin(2\bar{z}_c - \theta) \cong 1, \cos(2\bar{z}_c - \theta) \cong -\frac{1}{2\bar{z}_c}. \tag{57}$$

3.2.2. De Broglie-Bohm Quantum Trajectory

In region I, from (21) and (51) we have

$$\Theta_{(0)}(\phi_{rk}{}^{\mathrm{I}},\tau)=-\frac{k}{4}\frac{\left(\left|H_{1/2}{}^{(1)}\right|^2\right)'}{\left|H_{1/2}{}^{(1)}\right|^2}\sigma z^2 a^{-1}\phi_{rk}{}^{\mathrm{I}2}-\frac{1}{2}\int\frac{\frac{6}{\pi|\tau|}}{\left|H_{1/2}{}^{(1)}\right|^2}d\tau\ , \tag{58}$$

where $\left|H_{1/2}{}^{(1)}\ \sigma z^3/3\ \right|^2=6/\pi\sigma z^3$, and the prime in (58) denotes the derivative with respect to $\sigma z^3/3$. Substituting (58) into (13) and using $d\tau=dt/a$ and $z=k|\tau|\ =k/ah$ gives

$$\frac{d\phi_{rk}{}^{\mathrm{I}}}{d\tau}=\frac{3}{2}\frac{k}{z}\phi_{rk}{}^{\mathrm{I}}. \tag{59}$$

The general solution of (59) is

$$\phi_{rk}{}^{\mathrm{I}}=\hat{C}^{\mathrm{I}}z^{-3/2}. \tag{60}$$

On the other hand, in region II, from (23), (24), and (41)-(45) we again have

$$\varphi_{rk}{}^{\mathrm{II}}=\hat{C}^{\mathrm{II}}\left|H_{3/2}{}^{(1)}(z)\right|=\hat{C}^{\mathrm{II}}\sqrt{2/\pi}z^{-3/2}\ \ 1+z^2\ ^{1/2}. \tag{61}$$

Then, substituting (60) and (61) into the matching condition at $\bar{z}_c$ for $\phi_{rk}{}^{\mathrm{I}}$ and $\phi_{rk}{}^{\mathrm{II}}$

$$\phi_{rk}{}^{\mathrm{I}}\Big|_{\bar{z}_c}=\phi_{rk}{}^{\mathrm{II}}\Big|_{\bar{z}_c} \tag{62}$$

And using $\bar{z}_c>>1$, we obtain

$$\hat{C}^{\mathrm{II}}=\sqrt{\pi/2}\hat{C}^{\mathrm{I}}\bar{z}_c{}^{-1}. \tag{63}$$

Therefore, using (63) in (61) yields

$$\varphi_{rk}{}^{\mathrm{II}}=\hat{C}^{\mathrm{I}}\bar{z}_c{}^{-1}z^{-3/2}\ \ 1+z^2\ ^{1/2}. \tag{64}$$

Since for 3-dimensional ϕ_{rk} contains a factor $a^{3/2}$ which is proportional to $z^{-3/2}$, we use a field redefinition $u_{rk}\equiv a^{-3/2}\phi_{rk}$ and $a=(k/h)z^{-1}$ to rewrite (60) and (64) as

$$u_{rk}{}^{\mathrm{I}}=\left(\frac{k}{h}\right)^{-3/2}\hat{C}^{\mathrm{I}},\ u_{rk}{}^{\mathrm{II}}=\left(\frac{k}{h}\right)^{-3/2}\hat{C}^{\mathrm{I}}\bar{z}_c{}^{-1}(1+z^2)^{1/2}. \tag{65}$$

From (65) we see that for fixed k and $\bar{z}_c>>1$, as z decreases from $z=\bar{z}_c>>1$ to $z=1$, the scalar field value decreases $\bar{z}_c/\sqrt{2}$ times, i.e., there exists a transition in the time evolution of the quantum trajectory of scalar field.

To compare (48) with (65) for scalar field values, we set $\hat{C}^{\mathrm{I}}=\overline{C}^{\mathrm{I}}\bar{z}_c{}^{1/2}$ so that in region II the scalar field has almost the same evolution for the $s=1$ and $s=2$ cases. Then (65) becomes

$$u_{rk}{}^{\mathrm{I}}=\left(\frac{k}{h}\right)^{-3/2}\overline{C}^{\mathrm{I}}\bar{z}_c{}^{1/2},\ u_{rk}{}^{\mathrm{II}}=\left(\frac{k}{h}\right)^{-3/2}\overline{C}^{\mathrm{I}}\bar{z}_c{}^{-1/2}(1+z^2)^{1/2}. \tag{66}$$

From (48) and (66) we then find that for $b_1\approx b_2\sim 1,\ z_c\approx\bar{z}_c>>1$, the former is larger than the latter by a factor $\sqrt{z/z_c}$ in the early evolution as $z>z_c\approx\bar{z}_c$, while the former is approximately equal to the latter as z decreases from $z = z_c \approx \bar{z}_c >> 1$ to $z = 1$. Therefore, if we compare the trans-Planckian effects of both quartic and sextic corrections on the quantum trajectory of scalar field, the latter is smaller than the former.

4. Vacuum Energy and Cosmological Constant

Using the results of section 3, we proceed in this section to calculate the finite vacuum energy density, and use the back reaction constraint to address the cosmological constant problem. Notice that in the slow-roll approximation, the energy density of the scalar field is $\rho_\phi \cong V(\phi)$, where $V(\phi) = m^2\phi^2/2$ .Thus, the mean value of the vacuum energy density ρ_ϕ can be computed from the de Broglie-Bohm pilot-wave theory by defining a field configuration spatial average of a local magnitude weighted by $R_{(0)}{}^2$: [28]

$$\begin{aligned}\langle \rho_\phi \rangle &= \int_{-\infty}^{\infty} R_{(0)}{}^2 \frac{m^2\phi^2}{2} d\phi = \frac{m^2}{2}\frac{k^3}{2\pi^2} \int_{-\infty}^{\infty} \left|\psi_{rk(0)}(u_{rk},\tau)\right|^2 u_{rk}{}^2 du_{rk}\,, \\ &= \frac{m^2}{2}\frac{k^3}{2\pi^2}\frac{1}{2}\frac{1}{\mathrm{Re}(B_k(\tau)a^{-1})} a^{-3}\,, \end{aligned} \tag{67}$$

Where the scalar field is redefined by $u_{rk} \equiv a^{-3/2}\phi_{rk}$, and

$$\left|\psi_{rk(0)}(u_{rk},\tau)\right|^2 = a^{3/2}\frac{\sqrt{\mathrm{Re}(B_k(\tau)a^{-1})}}{\sqrt{\pi}} \exp\left(-\mathrm{Re}(B_k(\tau)a^{-1})\frac{u_{rk}{}^2}{a^{-3}}\right). \tag{68}$$

Here $\mathrm{Re}(B_k(\tau)a^{-1})$ denotes the real part of $B_k(\tau)a^{-1}$, and the factor $a^{3/2}$ in (68) appears through the normalization condition

$$\int_{-\infty}^{\infty} du_{rk}\left|\psi_{rk(0)}(u_{rk},\tau)\right|^2 = 1\,. \tag{69}$$

For the case $s=1$ and $b_1 > 0$, in region I, we have $\left|H_{3/4}{}^{(1)}\ \sigma z^2/2\ \right|^2 \approx 4/\pi\sigma z^2$ with $\sigma = z_c{}^{-1}$. Then, using $a = 1/h|\tau| = k/hz$ and (22) in (67), we find

$$\langle \rho_\phi \rangle_{s=1}^{\mathrm{I}} = \left(\frac{m^2}{2h^2}\right)\frac{1}{4\pi^2} z z_c h^4\,, \tag{70}$$

where $z_c = M/b_1^{1/2}h$. On the other hand, in region II, (24) can be expressed as

$$B_k{}^{\mathrm{II}}(\tau) = \frac{\dfrac{2}{\pi|\tau|}}{\left|H_{3/2}{}^{(1)}\right|_{md}^2} - i\frac{k}{2}\frac{\left(\left|H_{3/2}{}^{(1)}\right|_{md}^2\right)'}{\left|H_{3/2}{}^{(1)}\right|_{md}^2}\,, \tag{71}$$

with $\left|H_{3/2}{}^{(1)}\right|_{md}$ defined as

$$\left|H_{3/2}{}^{(1)}\right|_{md} \equiv \left\{\left(\left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2\right)\left|H_{3/2}{}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1{}^{\mathrm{II}}C_2{}^{\mathrm{II}*}\ H_{3/2}{}^{(1)}\ {}^2\right]\right\}^{1/2}, \tag{72}$$

where $\left|H_{3/2}{}^{(1)}(z)\right|^2 = \dfrac{2}{\pi z}\left(1+\dfrac{1}{z^2}\right)$ From (29), (33), and (34), we see that $\left|H_{3/2}{}^{(1)}\right|_{md}$ can be approximated by $\left|H_{3/2}{}^{(1)}\right|$ as z decreases from $z = z_c >> 1$ to $z = 1$. Then, using $a = 1/h|\tau| = k/hz$ and (24) in (67), we find

$$\left\langle \rho_\phi \right\rangle_{s=1}^{\mathrm{II}} = \left(\frac{m^2}{2h^2} \right) \frac{1}{4\pi^2} z^2 h^4 \left(1 + \frac{1}{z^2} \right). \tag{73}$$

From (70) and (73) we see that as z decreases from $z = z_c >> 1$ to $z = 1$, the vacuum energy density reduces $z_c^{\,2}/2$ times. Notice that this result can also be obtained from considering (48) about the time evolution of the quantum trajectory of scalar field at the end of subsection 3.1.2. We also notice that there is no back reaction problem if the energy density due to the quantum fluctuations of the inflaton field is smaller than that due to the inflaton potential, i.e.,

$$\left\langle \rho \right\rangle_{s=1}^{\mathrm{I}} (z > z_c) < V(\phi) \, . \tag{74}$$

Near the beginning of inflation at $z >> z_c$, the vacuum energy density due to the fluctuations of the inflaton field with $k < k_{\max}$ is expected to be [29]

$$\rho_{vac} = \frac{1}{2\pi^2} \int_0^{k_{\max}} \left(\frac{k}{a} \right)^4 \frac{dk}{k} = \frac{1}{8\pi^2} k_{phys(\max)}{}^4 = \frac{1}{8\pi^2} M^4, \tag{75}$$

where M is the cutoff in (16). Therefore, we evaluate $\left\langle \rho \right\rangle_{s=1}^{\mathrm{I}}$ in (70) at $z = z_c^{\,3} >> z_c$ so that

$$\left\langle \rho \right\rangle_{s=1}^{\mathrm{I}} (\, z = z_c^{\,3}) = \left(\frac{m^2}{2h^2} \right) \frac{1}{4\pi^2 b_1^{\,2}} M^4 \propto M^4 . \tag{76}$$

In the slow-roll approximation, using $V(\phi) \cong 3M_{Pl}^{\,2} h^2 / 8\pi$ and (76) in (74) gives the constraint on b_1 as $b_1 > \frac{1}{\sqrt{3\pi}} \left(\frac{m}{M_{Pl}} \right) \left(\frac{M^2}{h^2} \right)$. For $M \sim 10^{16}$ GeV, which is the energy scale during inflation as implied by the recent BICEP2 experiment [30, 31], we have $b_1 > 3.2 \times 10^{-3}$.

For the case $s = 2$ and $b_2 > 0$, in region I, we have $\left| H_{1/2}^{(1)} \;\; \sigma z^3 / 3 \right|^2 = 6 / \pi \sigma z^3$ with $\sigma = \bar{z}_c^{\,-2}$. Then, using $a = 1/h|\tau| = k / hz$ and (51) in (67), we find

$$\left\langle \rho_\phi \right\rangle_{s=2}^{\mathrm{I}} = \left(\frac{m^2}{2h^2} \right) \frac{1}{4\pi^2} \bar{z}_c^{\,2} h^4 , \tag{77}$$

where $\bar{z}_c = M / b_2^{\,1/4} h$. On the other hand, in region II, (24) can be again expressed as (71) with $\left| H_{3/2}^{(1)} \right|_{md}$ defined by (72). From (52), (56), and (57), we see that $\left| H_{3/2}^{(1)} \right|_{md}$ can be approximated by $\left| H_{3/2}^{(1)} \right|$ as z decreases from $z = \bar{z}_c >> 1$ to $z = 1$. Then, using $a = 1/h|\tau| = \; k / hz$ and (24) in (67), we find

$$\left\langle \rho_\phi \right\rangle_{s=2}^{\mathrm{II}} = \left(\frac{m^2}{2h^2} \right) \frac{1}{4\pi^2} z^2 h^4 \left(1 + \frac{1}{z^2} \right). \tag{78}$$

From (77) and (78) we see that as z decreases from $z = \bar{z}_c >> 1$ to $z = 1$, the vacuum energy density also reduces $\bar{z}_c^{\,2}/2$ times. Notice that this result can also be obtained from considering (65) about the time evolution of the quantum trajectory of scalar field in subsection 3.2.2. As before, there is no back reaction problem if

$$\langle\rho\rangle_{s=2}^{\mathrm{I}} < V(\phi). \tag{79}$$

Using $V(\phi) \cong 3M_{Pl}{}^2h^2/8\pi$ and (77) in (79) gives the constraint on b_2 as $b_2 > \frac{1}{9\pi^2}(\frac{Mm}{M_{Pl}h})^4$.

For $M \sim 10^{16}$ GeV, we have $b_2 > 1.4\times10^{-18}$.

Furthermore, we find that the vacuum energy density reduces significantly in the evolution from well before horizon to horizon exit, providing a possible mechanism for generating a small cosmological constant. Specifically, from (70) and (73) we can see that $\langle\rho_\phi\rangle_{s=1}$ changes as

$$\left(\frac{m^2}{2h^2}\right)\frac{1}{4\pi^2b_1^2}M^4 \to \left(\frac{m^2}{2h^2}\right)\frac{1}{4\pi^2b_1}M^2h^2 \to \left(\frac{m^2}{2h^2}\right)\frac{1}{2\pi^2}h^4 \tag{80}$$

when z changes as $z_c{}^3 \to z_c \to 1$. From (77) and (78) we also see that $\langle\rho_\phi\rangle_{s=2}$ changes as

$$\left(\frac{m^2}{2h^2}\right)\frac{1}{4\pi^2b_2^{1/2}}M^2h^2 \to \left(\frac{m^2}{2h^2}\right)\frac{1}{2\pi^2}h^4 \tag{81}$$

when z changes as $\bar{z}_c{}^3 \to \bar{z}_c \to 1$. Because $M >> h$ and $z_c \approx \bar{z}_c$ (for the usual parameters $b_1 \sim b_2 \sim 1$), $\langle\rho_\phi\rangle_{s=2}$ is always smaller than $\langle\rho_\phi\rangle_{s=1}$ in the early evolution when $z > z_c$, while $\langle\rho_\phi\rangle_{s=2}$ and $\langle\rho_\phi\rangle_{s=1}$ are approximately equal as z decreases from $z = z_c >> 1$ to $z = 1$. Notice that this result can also be obtained from comparing (48) and (66) about the time evolution of the quantum trajectory of scalar field at the end of subsection 3.2.2. Thus, if we consider quartic correction and neglect sextic correction in nonlinear dispersion relations, we find that when z decreases as $z_c{}^3 \to z_c \to 1$, the cosmological constant $\Lambda = 8\pi\rho_{vac}/M_{Pl}{}^2$ decreases as

$$1/\pi b_1^2 \quad M^4m^2/M_{Pl}{}^2h^2 \;\to\; (1/\pi b_1)\;\; M^2m^2/M_{Pl}{}^2 \;\to\; 2/\pi \quad m^2h^2/M_{Pl}{}^2 \tag{82}$$

Such a reduction of the cosmological constant happens during the inflationary era associated with the grand unification phase transition, at which the Hubble parameter is about 10^{14} GeV, and the energy scale during inflation is about 10^{16} GeV.

Since recent astronomical observations [32] indicate that the current vacuum energy density and cosmological constant are $\rho_{vac,0} \sim 2.5\times10^{-47}$ GeV ^{4} and $\Lambda_0 \sim 4.2\times10^{-84}$ GeV ^{2}, we suggest the possibility that a series of reductions similar to (82) could yield a small current value of the cosmological constant through cascade transition such as the electroweak, quark-hadron and current accelerating phase transitions in the evolution of the Universe.

5. Conclusion and Discussion

In the Schrödinger picture, we have considered the de Broglie-Bohm pilot-wave theory of a massless minimally coupled free real scalar field in de Sitter space. To investigate the possible effects of trans-Planckian physics on the quantum trajectory of the vacuum state of scalar field, we considered the CJ type dispersion relations with quartic or sextic correction. Through de Broglie's first-order dynamics, we find that there exists a transition in the evolution of the quantum trajectory from well before horizon exit to horizon exit, providing a possible mechanism for generating a small cosmological constant. Moreover, we find that if we compare the trans-Planckian effects of both quartic and sextic corrections on the quantum trajectory, the latter is much smaller than the former.

We also calculate explicitly the finite vacuum energy density due to fluctuations of the inflaton field, and show how the cosmological constant reduces during the slow-roll inflation at the grand unification phase transition. Then we suggest the possibility that a series of similar reductions such as electroweak, quark-hadron and current accelerating phase transitions could yield a small current value of the cosmological constant in the evolution of the Universe.

Finally we note that (75) is calculated in the usual quantum theory, in which the standard quantum equilibrium state in the context of de Broglie's dynamics is such that an ensemble of inflaton fields with the initial wave functional $\psi(\phi_{rk},\tau_0)$ has initial field configurations $\phi_{rk}(\tau_0)$ that are distributed according to the Born rule, with a density of probability $P(\phi_{rk},\tau_0)=|\psi(\phi_{rk},\tau_0)|^2$ in configuration space at τ_0. The effects of possible non equilibrium ensemble distributions $P(\phi_{rk},\tau_0)\neq|\psi(\phi_{rk},\tau_0)|^2$ at the Planck scale or during the inflation may be shown through the inclusion of nonequilibrium factor $\zeta(k)$ for each mode [12]. This correctional factor appears in the power spectrum for the inflaton fluctuations or the curvature perturbations, and in principle can be constrained by current observational cosmic microwave background data. As to the cosmological constant, the presence of similar correctional factors in (75) may change its value during inflation, but does not change essentially its reductive mechanism in (82). Therefore, it is expected that the prediction (82) applies not only for quantum nonequilibrium state with $\zeta(k)\neq 1$, but also for quantum equilibrium state with $\zeta(k)=1$.

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AUTHOR'S BIOGRAPHY

Jung-Jeng Huang received M. S. degree from the Graduate Institute of Physics, National Taiwan Normal University, Taipei, Taiwan, in 1993. He is currently an Associate Professor in the Department of Mechanical Engineering, and the Director of the Committee of Physics Teaching Research, Ming Chi University of Technology, Taiwan. His current research interests include gravitation, cosmology, particle physics, and optoelectric physics.